Effectiveness of Tridiagonal Path Dependent Option Valuation in Weak Derivative Market Environment

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Abstract

Accurate option price path is needed for risk management and for financial reporting as the accounting standards insist on mark to market value for derivative products. The binary models - Black Scholes model and Longstaff methods provide an insight on option pricing, but they are seldom validated with real data. Most of the research studies demonstrate the validity through algebra or by solving partial differential equations with strong market data. As these computations are tedious, and they are rarely applied in weak and incomplete markets. In this paper, we tested the actual values of options of Tata Consultancy Services whose shares and options are actively traded on the NSE, and applied the Crank Nicolson central difference method for estimating the path of the option pricing with five exercise prices, two out of money, at the money, and two in the money contracts of call and put options at three volatilities. The actual option values were compared with the forecasted option values and the errors were estimated. The plots of actual option values and the forecasted values produced by the central difference method converged excellently, producing minimum sums of squared error. Even in the weak and incomplete markets, the Crank Nicolson method worked well in producing the price path of options. This algorithm will be useful for hedging decisions and also for accurate forecasting and accounting reporting.

Keywords: **call and put options, Crank - Nicolson method, derivatives, partial differential equation, path dependent**

JEL Classification: C3, C4, G3, M1

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Recent trends in accounting standards mandate the companies to report off balance sheet items, like option contracts on balance sheet at market price or fair price, whichever is less (International Accounting Standards Board, 2003). Any differences between issue price and market value are to be debited to the income statement immediately except the specific hedge designated instruments, which fall under hedge accounting. Research in financial products (especially the derivatives, which are like betting contracts, their pricing mechanisms, risk quantification, etc.) helps the fund managers and also the regulators. This article examines the suitability of the Western market models developed for efficient markets - whether they work in weak incomplete markets like the Indian stock market with actual data. An important benchmark for calibration is needed in all spheres of capital markets to control when deviation arises. The investors, speculators, and fund managers will welcome such benchmarks for their decision making. The major contribution of this paper is that it

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gives an accurate price path which will be useful in reporting derivative products in financial statements correctly. Apart from adding to the existing literature, this article will provide an impetus for further research in incomplete capital markets.

Herd behaviour dominates the investment and reinvestment decisions among investors and speculators, especially in weak markets. Investors rarely analyze the economy, industry, and companies before investing in financial assets. It is all the more true for derivative investments like options. Our interest is to test whether the sophisticated option valuation models and techniques developed for strong markets are suitable for valuing options traded in developing and incomplete markets like India. Does the forecast option price path of option contracts based on sophisticated algebraic models converge with the actual price path? What level of error these models produce when the full path is considered in absolute terms and relative terms?

The Indian market cannot be compared with strong markets for three main reasons. Firstly, this market is weak as the investors rarely analyze the fundamental data before making investment decisions (Jayakumar, David, John, & Dawood, 2012). Secondly, they play on market sentiments and mostly indulge in speculation. In addition, this speculation is only carried out by the elite group of investors, and it does reach the common investors. This is due to the sophisticated nature of these derivative financial products. They baffle even the experienced financial engineers sometimes. Thirdly, investor education and data availability, all hinder the trading efficiency of these markets (Mallikarjunappa & Dsouza, 2013). The Indian capital market introduced online trading progressively, which promises fair, transparent, and fraudless investment dealings. Many investors and speculators who were operating using traditional methods were forced to change their style to grab this opportunity. This was another impediment as the old folks' mindset was not ready to accept these changes as they were very comfortable with the traditional methods. Despite these difficulties, the stock market and the derivative markets continue to grow strongly in India (Nagaraj, 1996).

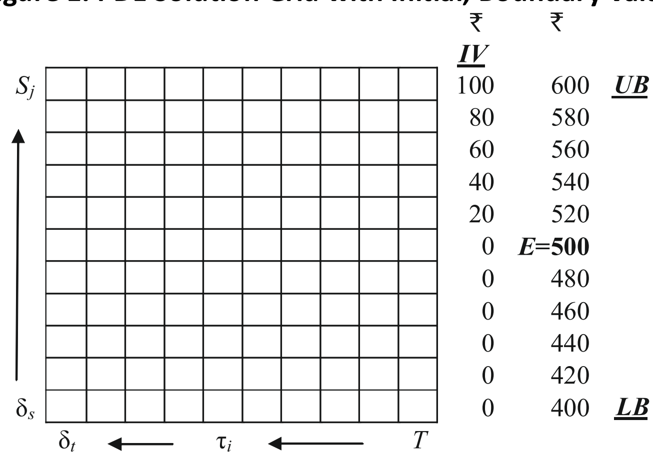
Globally, most of the stock markets are either weak form efficient or semi strong form efficient. The models developed for strong efficient markets in option pricing area have been mostly demonstrated analytically and rarely have been tested empirically (Boyle & Potapchik, 2008). Two difficulties have been proposed by the researchers for this. Firstly, the models are too analytical with a strong dose of algebra, where investors and researchers struggle and grapple to understand them (Shu, Gu, & Zheng, 2002). Secondly, they pointed out the non availability of relevant data, software, and computer memory that is needed to crunch voluminous data to get meaningful information for decision making, especially in hedging large portfolios. These large portfolios are built not only by institutional investors, but also by private fund managers and investment companies. Black Scholes and Longstaff methods provide models to value options, but assume that the returns are normally distributed and apply probability based on normal distribution, and this produces the smiles in option values (Düring, Fournié, & Jüngel, 2003). The Longstaff model is suitable for American type options, which could be exercised anytime during the life of the option contract (Bayraktar & Xing, 2009). This model applies the regression technique by taking option payoff as a dependent variable and quadratic, cubic, and higher power returns as independent variables. These methods are not applied in developing markets because they still follow the European style option contracts, mainly to avoid confusion among the investors.

The main objective of this paper is to test Crank - Nicholson (CN) central difference model's efficiency in predicting an option's value path. We chose Tata Consultancy Services (TCS) option contracts in call and put form for testing in this article. TCS is an active company not only in the stock and derivative markets, but also in business. The price path created by the CN model will be compared with the actual price path for finding their convergence or divergence. The deviation of forecasted price path from the actual price path will be assessed by using root mean squared errors (RMSE).

Methodology

Finding an option price path in a closed form is challenging not only because of tedious number crunching, but

Figure 1. PDE Solution Grid with Initial, Boundary Values



E = Exercise price; IV Initial values; UB = Upper Boundary; LB = Lower Boundary; δ_s = share price increment; S_j = Maximum Share price; δ_t = one day; τ_i = position of the day; T = Maturity; ₹ = Rupees

also because of several methods with different basic assumptions (Ameur, Breton, Karoui, & L'Ecuyer, 2007). The explicit, implicit, and the Crank Nicholson (central difference method) give a wider choice for option valuation, each with their own merits and demerits (Ehrhardt & Mickens, 2008). Nevertheless, the main objective is solving the partial differential equation to get a path of option values. In the money, at the money, and out of money contracts' price paths may differ substantially as the expectation of investors and speculators differ. Moreover, the price path of call and put contracts are to be in equilibrium, else the arbitrage profits will emerge. To generate the price path of options, researchers use the parabolic heat equation, which closely resembles with partial differential equation. The primary goal is to get a price path with minimum errors. A grid system is used in physics to solve a heat equation, which is a parabolic partial differential equation (PDE) (d'Halluin, Forsyth, Vetzal, Labahn, 2001). To solve PDE initial values, upper and lower boundary values are to be provided. Final (initial) values are the maturity payoff; upper and lower values are the maximum and minimum share prices that prevailed during the life of the option contract. The Figure 1 explains this concept and values clearly with imaginary values.

The Lattice Method

Solving PDE is tedious because it is not like simultaneous equations. Numerical values could be obtained by any one of the three methods, that is, explicit, implicit, and CN central difference. The initialization of values is as given in the Figure 1. A backward iterative procedure is carried out to get subsequent values from maturity to issue date.

$$\text{Option payoff} = \begin{cases} \text{maximum of ((spot - exercise),0),} & \text{Call} \\ \text{maximum of ((exercise-spot),0),} & \text{Put} \end{cases} \qquad (1)$$

The Figure 1 gives the grid for option values (Clarke & Parrott, 1999) that are to be computed at various times and underlying price intervals. In the extreme right, the expected share prices and the strike price *(E)* are given. The step value *(s_j)* of share price is taken as ₹ 20. In real valuation, a step value of ₹1 is considered to get an accurate value path. The adjacent left column gives the payoff of call options at maturity computed by using equation (1). The put option payoffs will be in reverse order.

Table 1. Inputs for Partial Differential Equation

Contract	Call - ₹	Put - ₹
Exercise price	500	500
Initial Condition on maturity	$u(x,91)$ = max (0, S-E)	$u(x,91)$ = max (0, E-S)
Lower Boundary share price	$u(x,0) = 400$	$u(x,0) = 400$
Upper Boundary share price	$u(11,y) = 600$	$u(11,y) = 600$
Upper Boundary option value	$u(11,y) = 100$	$u(11,y) = 0$
Lower Boundary option value	$u(x,0) = 0$	$u(x,0) = 100$

u(x,y) denotes the co-ordinate values in the grid

The exercise price, lower and upper boundary values are arbitrary & can be replaced with actual values of a company.

The lower boundary value for the share price is ₹ 400, and the upper boundary value is ₹ 600. These prices are the minimum and maximum expected prices within the period of contract life of three months. The current spot price of a share is taken as the exercise price so that half of the prices will be in the upper boundary area, and another half will be in the lower boundary area. This will take care of in the money, at the money, and out of money contracts. If the contract is in the money, it produces payoffs. If the contract is at the money or out of money, there is no value for the contract, and hence, they show zero values. The x - axis represents the time, and δ_t is considered to be one day (in terms of year 1/365). Unlike a heat equation, in option pricing, time decreases from maturity date one by one and reaches the option issue date. At the time of maturity of the option, only intrinsic values are left and there is no time value as the contract has already matured. The Table 1 presents the list of parameters in a nutshell. All these values are pertinent to TCS option contract, and they are applied in solving the PDE (2).

To get the option value at every lattice point, the following PDE is to be solved, which is based on the Black - Scholes method.

$$\frac{\partial v}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 v}{\partial s^2} + rS\frac{\partial v}{\partial s} - rV = 0 \qquad (2)$$

where,
S = Share value,
σ = Annual standard deviation of returns,
r = Risk free rate per annum,
V = Value of option.

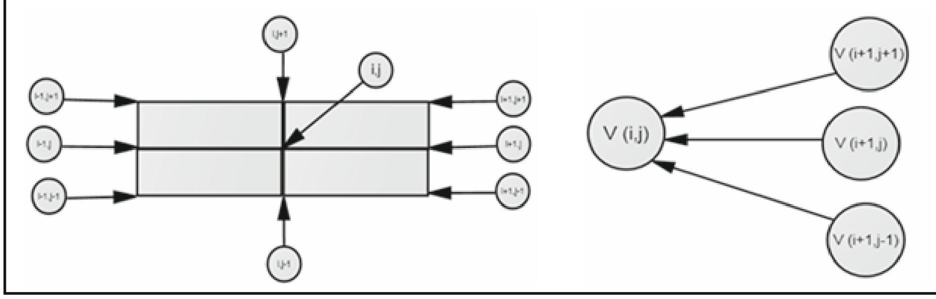
There are four sections on the left side of formula (2). They denote three motions, and the fourth item is the previous day's option value. This equation explains the drift, diffusion, and convection motion pollutants in rivers and streams (Bank, Wan, & Qu, 2005). The options in finance also have these three motions. The first section is theta, which captures the drift; the second portion is gamma, which quantifies diffusion ; and the third section is delta, which quantifies the convection.

Solving the PDE given in formula (2) numerically on each day is cumbersome and challenging, and may be solved by using any one of the following methods : explicit method, implicit method, and the Crank - Nicholson central difference method. Each method is explained as follows with their merits and weaknesses.

The Explicit Method

The explicit method works on the principle of interpolation. The explicit method is simple and easy when compared to implicit and CN methods (Jiwu, Yonggeng, Xiaotie, & Weimin, 2005). Computing the values of theta, gamma, and delta are the main thrusts of the path dependent option pricing, and it is to be done each day in a

Figure 2. The Lattice Points that are to be Considered to Compute a Single Value at *(i , j)*



recursive manner from maturity to origin backwards. The grid interpolation technique followed by the explicit method is as follows :

Leaving the upper and lower boundary values, all values at inner lattice points for each day at several share prices are to be computed. For a lattice point, there are eight neighbouring points as depicted in the Figure 2. The Figure 2 depicts the lattice points stationed at various positions and their indices to identify easily. For implicit and explicit methods, these points are enough. But for the CN method, six points are needed on the X- axis and three points are needed on the Y- axis as illustrated in the first part of the Figure 2.

To solve the PDE, the differential equations are be transformed into difference equations as depicted in the Table 2. The Table 2 describes the dynamics of option pricing in relation to the underlying pricing in three terms. The delta is the drift ; gamma is the diffusion ; and the theta is convention, as compared to water flows in a stream. If chemicals are dumped into a stream, they will go along with water (drift), will spread in water (diffusion), and some chemicals will be deposited (convection) in the river bed. To get the total effect of the chemical spread, the three are to be combined. With this analogy, the option prices are computed taking water as the underlying and the chemical as the option.

Combining the delta, gamma, and theta (the three parts), the difference equation becomes :

$$\left[\frac{V_i^{\,j+1} - V_i^{\,j}}{\delta t}\right] + \left[\left(r - \frac{1}{2}\sigma^2\right)\frac{V^{j+1}_{\ i+1} - V^{j-1}_{\ i+1}}{2\delta x}\right] + \left[\left(\frac{1}{2}\sigma^2\right)\frac{V^{j+1}_{\ i+1} - 2V^{j}_{\ i+1} + V^{j-1}_{\ i+1}}{\delta x^2}\right] - [rV^{j}_{\ i}] = 0 \qquad (3)$$

In explicit backward, the finite difference scheme taking *j* as the step size of space variable (Share price) and taking *i* as the step size of time variable $i, j = 1, 2, \ldots.$

In other words, it is :

$$\text{Theta} + \text{Delta} + \text{Gamma} - \text{Current value} = \text{zero} \qquad (4)$$

Putting it in another way, theta is equal to (change in value due to time) current value minus delta and gamma.

Table 2. Drift, Diffusion, and Convection Formulae for Option Pricing in Difference Form

Item	Difference Equation
Delta	$rS\,\frac{\partial V}{\partial x} = rS\left[\frac{V^{j+1}_{\ i+1} - V^{j+1}_{\ i+1}}{2\delta x}\right]$
Gamma	$\frac{1}{2}\,\sigma^2 S^2\,\frac{\partial^2 v}{\partial x^2} = \frac{1}{2}\,\sigma^2 S^2\left[\frac{V^{j+1}_{\ i+1} - 2V^{j}_{\ i+1} + V^{j-1}_{\ i+1}}{\delta x^2}\right]$
Theta	$\frac{\partial V}{\partial t} = \left[\frac{V^{j}_{\ i+1} - V^{j}_{\ i}}{\delta t}\right]$

Figure 3. Coefficient, Value, and Constant Matrices

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ c & b & a & 0 & 0 & 0 \\ 0 & c & b & a & 0 & 0 \\ 0 & 0 & c & b & a & 0 \\ 0 & 0 & 0 & c & b & a \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} UB \\ V_{..} \\ V_{..} \\ V_{..} \\ V_{..} \\ \ldots \\ \ldots \\ LB \end{pmatrix} = \begin{pmatrix} UB \\ pf_1 \\ pf_2 \\ pf_{..} \\ pf_{..} \\ \ldots \\ \ldots \\ LB \end{pmatrix}$$

where,
LB = lower boundary ; *UB* = upper boundary; *pf* = payoff ;
V = value of option

The option values are computed by interpolation one by one - first vertically for all underlying values and then horizontally for each day recursively (Boyle & Tian, 1998).

The payoff given at maturity will determine the values of options one day backward and so on till it reaches the origin. This is simple to implement, numerically non-tedious, as it is only an interpolation technique. However, convergence is the problem (Wang & Wang, 2006). Unless the ratio ($\delta t/j^2$) is less than 0.5, the convergence may not be attained. The results are reliable in the first order accurate in time direction and second order accurate in the space (share price) direction. This method quickly becomes unstable as the errors grow fast (Kwok & Lau, 2001a). Hence, this method is not reliable (Kudryavtsev & Levendorskiĭ, 2009).

The Implicit Method

When compared to the explicit method, the implicit method is complicated, but it converges quickly with little dampening effect. This method is complicated because it solves several PDEs simultaneously for each day in the reverse order recursively. The number of equations grows in multiples when small values are taken in the time and price directions. The UB and LB values are to be preserved while working with time to get accurate inner lattice values.

These PDEs could be solved by Gaussian inversion, but time and memory could be saved if the tridiagonal matrix method is applied (Gilli, Këllezi, & Pauletto, 2002). The steps involved in designing and solving the tridiagonal matrix are illustrated below. If equation (2) is rearranged, the coefficients will be as follows :

$$a = 0.5rj\,\delta t - 0.5\,\sigma^2 j^2 \delta t \quad (5)$$
$$b = 1 + \sigma^2 j^2\,\delta t + r\delta t \quad (6)$$
$$c = -0.5rj\delta t - 0.5\,\sigma^2 j^2\,\delta t \quad (7)$$

where,
j = Share value,
σ = Annual standard deviation of returns,
r = Risk free rate per annum,
V = Value of option,
δt = One day

To determine the values at interior grid points, we cannot work backward as done in the explicit method. Here, the grid values are to be computed simultaneously by solving a system of linear equations. For n unknown values, there will be $n+2$ equations, including the two boundary values, which are not to be changed at each iteration when we move forward. The Figure 3 shows the tridiagonal coefficient matrix, the value column vector, and the constant column vector.

The Figure 3 matrix could be solved by Gaussian inversion, which requires more memory and time. A closer observation will show that the Figure 3 coefficient matrix is tridiagonal, which could be solved by the matrix division method. This saves memory and time. In addition, the results arrived by this implicit method converge with minimum error. The first iteration, matrix division gives the option values of the previous day as we work from maturity to origin. The newly computed values will be considered as payoff for the next iteration, and the third column values computed and so on. In this method, the convergence problem is tackled, to some extent, but still, to get refined values, the ratio of time to space should be less than 0.5, which increases the number of PDEs to be solved.

The Crank - Nicolson Method

The explicit and implicit methods give values of options in the interior lattice points single order in time and second order in space (share price). This is a major weakness. To overcome the above problem, the Crank - Nicolson (CN) method is used, which is stable always and converges second order in time also (Curien, Jean-Noel, & Stephane, 2003). This method works on the principle of central difference. The initial values and the boundary values are applied as in the implicit method. The CN PDEs are solved in two steps (Kwok & Lau, 2001b). The tridiagonal matrix on the right side is scaled by payoff values at maturity first. In step two, the left matrix and the scaled payoff values are taken as coefficient and values and the PDEs are solved to get all option values simultaneously as done in the implicit method (Dyrting, 2004). By iteration, all values are computed.

If option values of n^{th} day are to be computed from the $n+1^{th}$ day, the following equation is to be solved (right to left) :

$$\left[\frac{V_i^{j+1} - V_i^{j}}{\delta t}\right] + \left[\left(r - \frac{1}{2}\sigma^2\right)\frac{V^{j+1}_{i+1} - V^{j-1}_{i+1}}{2\delta x}\right] + \left[\frac{1}{2}\sigma^2 \frac{V^{j+1}_{i+1} - 2V^{j}_{i+1} + V^{j-1}_{i+1}}{\delta x^2}\right] - [rV^{j}_{i}] = 0 \qquad (8)$$

If option values of $n+1^{th}$ day are to be computed from n^{th} day, the following equation is to be solved (left to right) :

$$\left[\frac{V_i^{j+1} - V_i^{j}}{\delta t}\right] + \left[\left(r - \frac{1}{2}\sigma^2\right)\frac{V^{j+1}_{i} - V^{j-1}_{i}}{2\delta x}\right] + \left[\frac{1}{2}\sigma^2 \frac{V^{j+1}_{i} - 2V^{j}_{i} + V^{j-1}_{i}}{\delta x^2}\right] - [rV^{j}_{i}] = 0 \qquad (9)$$

After averaging and rearranging Equation 8 and Equation 9, the following equation emerges :

$$\left(\frac{\sigma^2 j^2}{2} - \frac{rj}{2}\right)V_i^{j-1} + \left(\frac{-2}{\delta t} - \sigma^2 j^2 - 2r\right)V_i^{j} + \left(\frac{\sigma^2 j^2}{2} + \frac{rj}{2}\right)V_i^{j+1}$$
$$=$$
$$\left(\frac{rj}{2} - \frac{\sigma^2 j^2}{2}\right)V^{j-1}_{i+1} + \left(\frac{-2}{\delta t} + \sigma^2 j^2\right)V^{j}_{i+1} + \left(-\frac{rj}{2} - \frac{\sigma^2 j^2}{2}\right)V^{j+1}_{i+1} \qquad (10)$$

After several rearrangements, the coefficients emerge as follows. For easy representation, they are given as follows :

Figure 4. Coefficient, Value, and Constant Matrices (RHS)

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ c & 1+b & a & 0 & 0 & 0 \\ 0 & c & 1+b & a & 0 & 0 \\ 0 & 0 & c & 1+b & a & 0 \\ 0 & 0 & 0 & c & 1+b & a \\ .... & .... & .... & .... & .... & .... \\ .... & .... & .... & .... & .... & .... \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} UB \\ pf_1 \\ pf_2 \\ pf_{..} \\ pf_{..} \\ .... \\ .... \\ LB \end{pmatrix} = \begin{pmatrix} NUB \\ spf_1 \\ spf_2 \\ spf_{..} \\ spf_{..} \\ .... \\ .... \\ NLB \end{pmatrix}$$

Note : The Right Matrix is used to compute the scaled payoffs

Note : *pf* = payoff; *spf* = scaled payoff; *npf* = new payoff; *UB* = upper boundary; *LB* = lower boundary; *NUB* = new upper boundary; *NLB* = new lower boundary

Figure 5. Coefficient, Value, and Constant Matrices (LHS)

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -c & 1-b & -a & 0 & 0 & 0 \\ 0 & -c & 1-b & -a & 0 & 0 \\ 0 & 0 & -c & 1-b & -a & 0 \\ 0 & 0 & 0 & -c & 1-b & -a \\ .... & .... & .... & .... & .... & .... \\ .... & .... & .... & .... & .... & .... \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} UB \\ npf_1 \\ npf_2 \\ npf_{..} \\ npf_{..} \\ .... \\ .... \\ LB \end{pmatrix} = \begin{pmatrix} NUB \\ spf_1 \\ spf_2 \\ spf_{..} \\ spf_{..} \\ .... \\ .... \\ NLB \end{pmatrix}$$

Note : The Left Matrix is used to compute the expected option values

Note : *pf* = payoff; *spf* = scaled payoff; *npf* = new payoff; *UB* = upper boundary; *LB* = lower boundary; *NUB* = new upper boundary; *NLB* = new lower boundary

$$a = \frac{\delta t}{4} * \sigma^2 j^2 - rj \, ; \, b = -\frac{\delta t}{2} * \sigma^2 j^2 + r \, ; \; c = \frac{\delta t}{4} * \sigma^2 j^2 + rj \quad (11)$$

The Figure 4 illustrates the right hand side matrices which are used to get the scaled payoffs. The coefficient matrix is multiplied with the payoff column vector *(pf)* to get scaled payoffs *(spf)*. The Figure 5 portrays the left hand side matrices which are used to compute the option values by using the central difference method. The scaled payoffs computed in Figure 4 are used as the pseudo option values while solving the PDE and real option values are arrived in the central column vector.

The PDEs are solved backward in time as in the implicit method, and compute the new values in all lattice points simultaneously (Zhang, 1997). The newly computed values for the n-1th day is treated as the new IVs for the n-2th day and the coefficient matrix again recomputes the new values for the next time step, and this iteration process is repeated till the time it reaches t_0. The Crank - Nicolson method has several merits when compared to explicit and implicit methods. In addition, this algorithm is second order convergent in time step also. Thus, it gives more accurate values of options at lattice points.

Table 3. Descriptive Statistics of TCS Call Option

	TCS Price	1200 LTP	1200 STP	1250 LTP	1250 STP	1300 LTP	1300 STP	1350 LTP	1350 STP	1400 LTP	1400 STP
Mean	1285.72	80.27	116.11	54.73	78.09	32.11	48.03	16.72	27.43	7.03	14.61
Std Dev	59.27	69.47	40.83	49.99	34.57	34.97	26.09	20.60	17.39	9.91	9.22
Co Vari	0.05	0.87	0.35	0.91	0.44	1.09	0.54	1.23	0.63	1.41	0.63
Skewness	0.61	0.26	1.03	0.46	1.13	1.07	1.16	1.38	1.05	1.55	0.94
Kurtosis	-0.48	-1.05	0.33	-0.92	0.45	0.29	0.81	1.17	0.74	1.68	0.41
Minimum	1185.90	0.00	59.95	0.00	35.70	0.00	6.70	0.00	0.70	0.00	0.20
Maximum	1417.70	215.60	221.65	153.00	171.90	123.60	123.60	76.95	77.30	37.80	37.35

LTP= Last traded price; STP= Settlement Price; Std Dev = Standard deviation; Co Vari= Coefficient of Variation

Data

To test the efficiency of the Crank - Nicolson method in the Indian derivative market, we chose TCS's call and put option contracts' values which matured in April 2013. This research was conducted during 2013. In total, we took 10 TCS contracts - two in the money, two out of money, and one at the money each in call and put. These contracts had a life of 91 days. From the National Stock Exchange of India (NSE) website, we downloaded data for the above contracts including underlying value.

The underlying value is a column vector determined by the minimum and maximum values in increments of ₹ 1. As discussed earlier, the strike prices are close to the spot price in case of at the money contract and for out of money and in the money contracts, ₹ 50 is given as step values.

The NSE of India operated only for 65 days during the study period. The time value for the option contracts decreases on a daily basis as the time to maturity declines steadily, whether the stock exchange operates or not. The finite difference method will give accurate values when the time step increases uniformly. To overcome this problem of an unequal time interval in the real underlying data, we inserted the same Friday value for Saturday and Sunday on the assumption that the share prices did not change as there is no transaction on these days. In addition, the prices change only on Monday morning when the stock market opens, till such time, the share prices are in hibernation.

Another way of tackling this problem is to fill up the gaps of Saturdays and Sundays with the time-series share price mean values. This will avoid two problems encountered by the previous computation. Firstly, it will not alter the mean and standard deviation of the series, and at the same time, it will give periodical data for PDEs. However, this method also has a weakness in the form of different prices, which will appear in the middle, affecting the sequence of prices. Therefore, to get smoothness, we tried the former method.

After filling the data for Saturdays and Sundays, we computed the returns by differencing them and computed mean and volatility (standard deviation). The volatility of underlying returns is the subject matter of many research articles. The volatility smile and moving unstable volatility were the two principal branches of research in the past. We took three different volatilities as low, medium, and maximum to assess the efficiency of the CN method. With the annual base lending rate fixed by the Reserve Bank of India and 0.0027 years (1/365, one day) as the time increment, we wrote a MATLAB program to compute the lattice of option prices from day 91 to option issue date in reverse order for each share price, that is, from minimum to maximum.

Analysis, Results, and Discussion

The underlying prices of TCS and five option values at different strike prices were downloaded from the National Stock Exchange of India. They were from ₹ 1200 to ₹ 1400 in step values of ₹ 50. Their descriptive statistics are as

Table 4. Descriptive Statistics of TCS Put Option

	TCS Price	1200 LTP	1200 STP	1250 LTP	1250 STP	1300 LTP	1300 STP	1350 LTP	1350 STP	1400 LTP	1400 STP
Mean	1,285.72	4.37	16.95	6.00	29.85	9.69	49.46	12.45	77.88	19.73	113.50
Std Dev	59.27	7.31	15.61	9.15	24.33	14.13	33.62	18.86	41.60	32.53	47.75
Skewness	0.61	2.16	0.54	1.63	0.28	1.63	0.08	1.20	-0.20	1.42	-0.45
Kurtosis	-0.48	3.94	-0.93	1.39	-1.27	2.05	-1.37	-0.10	-1.15	0.61	-0.75
Minimum	1,185.90	0.00	0.10	0.00	0.65	0.00	2.05	0.00	5.50	0.00	16.25
Maximum	1,417.70	27.70	57.20	29.90	85.05	52.40	118.65	61.85	157.00	107.20	199.00

LTP= Last traded price; STP= Settlement Price; Std Dev = Standard deviation; Co Vari= Coefficient of Variation

follows. The settlement prices (STP) were taken from the date of issue of contract while the last traded prices (LTP) were available from the date when contracts were purchased by the investors. The LTPs were zero before the contracts were traded. The Tables 3 - 7 describe the descriptive statistics of the call and put options to understand the property and pattern of the option prices that were estimated.

The Table 3 and Table 4 present the descriptive statistics of actual option prices, while Table 5, Table 6, and Table 7 depict the estimated option prices descriptive statistics of both call and put contracts at three different volatilities of 10%, 30%, and 50%. The mean and standard deviations of the prices show the distribution of the price series, while the coefficient of variation indicates the spread of the prices in relative terms. All LTPs show greater variance as they are the real prices of traded options. The STPs' variation is approximately half of the LTPs variation. LTPs are theoretical prices based on a certain model, like the Black - Scholes model, while the LTPs are based on the market forces. The coefficient of variation increases when the strike prices increase both for the LTPs and STPs.

The Table 3 gives the actual option price descriptive statistics at 10 different price levels for TCS call contracts. The skewness and kurtosis show the shape of the distribution and also the thickness of the tails. A skewness value of zero indicates normal distribution. LTPs show smaller skewness values, while the STPs show higher skewness values. In the money contracts are close to normal, while at the money and out of money contracts show values larger than one, which indicates right skewed distributions. The kurtosis values for in the money contracts are negative, which indicate fat tails in their distributions, while at the money and out of money contracts show thin tails. The maximum and minimum prices indicate the range within which these prices moved during the life of the contracts.

The mean prices and other useful parameters of TCS put option contracts are given in the Table 4 at 10 different strike price levels. The LTP means are far less than the STP prices. The STPs are quoted from the day of issue, but the last traded price is quoted when the investors buy the contracts else quoted as zero, and this is the reason for their lower average price. Similar behavior could be observed in standard deviation also. All skewness values are closer to zero, which indicates that the values are normally distributed. The in the money put contracts show a slightly negative skew in STPs. The kurtosis values are all negative for STPs, which indicate flat tails and platykurtic properties of STPs.

The Table 5 depicts the descriptive statistics of both call and put options estimated at 10% volatility for five TCS underlying market prices, which are closer to the strike prices. The Crank - Nicolson prices, though computed for all price steps at 10% volatility, are filtered by matching the real price of TCS and the lattice price to select the appropriate call and put option values from the lattice of option values. The mean values computed by the CN method are closer to the LTPs and STPs for both the call and put options. However, the skewness values of call contracts increase sharply and even reach a value of 4.02 when the contract is out of money. However, for put contracts, the skewness values decrease from 8.28 to -0.51. It seems the CN value distributions are normal only when the contracts are at the money, but they are skewed when the contracts are out of money and in the money. The kurtosis values increase steadily for call contracts and decrease for put contracts. But they show very high

Table 5. Descriptives - Crank Nicolson Forecasted Option Values at a Volatility of 0.10

	1200		1250		1300		1350		1400	
	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
Mean	86.32	0.24	43.85	7.78	18.95	32.86	6.22	70.16	0.81	114.78
Std Dev	58.91	1.66	51.03	14.78	33.97	33.16	16.74	48.54	3.35	57.68
Skewness	0.64	8.28	1.04	2.08	1.75	0.61	2.75	-0.10	4.02	-0.51
Kurtosis	-0.49	72.19	-0.08	3.73	1.87	-0.84	6.49	-1.18	14.72	-0.70
Minimum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Maximum	217.22	15.00	167.22	65.00	117.21	115.00	67.21	165.00	17.20	215.00

Table 6. Descriptives - Crank Nicolson Forecasted Option Values at a Volatility of 0.30

	1200		1250		1300		1350		1400	
	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
Mean	88.41	0.24	45.43	7.77	19.62	32.41	6.67	69.98	0.94	114.78
Std Dev	58.94	1.65	51.16	14.63	34.41	33.15	16.96	48.68	3.51	58.33
Skewness	0.60	8.38	1.02	2.15	1.70	0.66	2.66	-0.05	3.90	-0.47
Kurtosis	-0.52	73.77	-0.15	4.07	1.67	-0.75	6.01	-1.18	13.86	-0.75
Minimum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11
Maximum	217.29	15.00	167.27	65.00	117.26	115.00	67.24	165.00	17.34	215.00

Table 7. Descriptives - Crank Nicolson Forecasted Option Values at a Volatility of 0.50

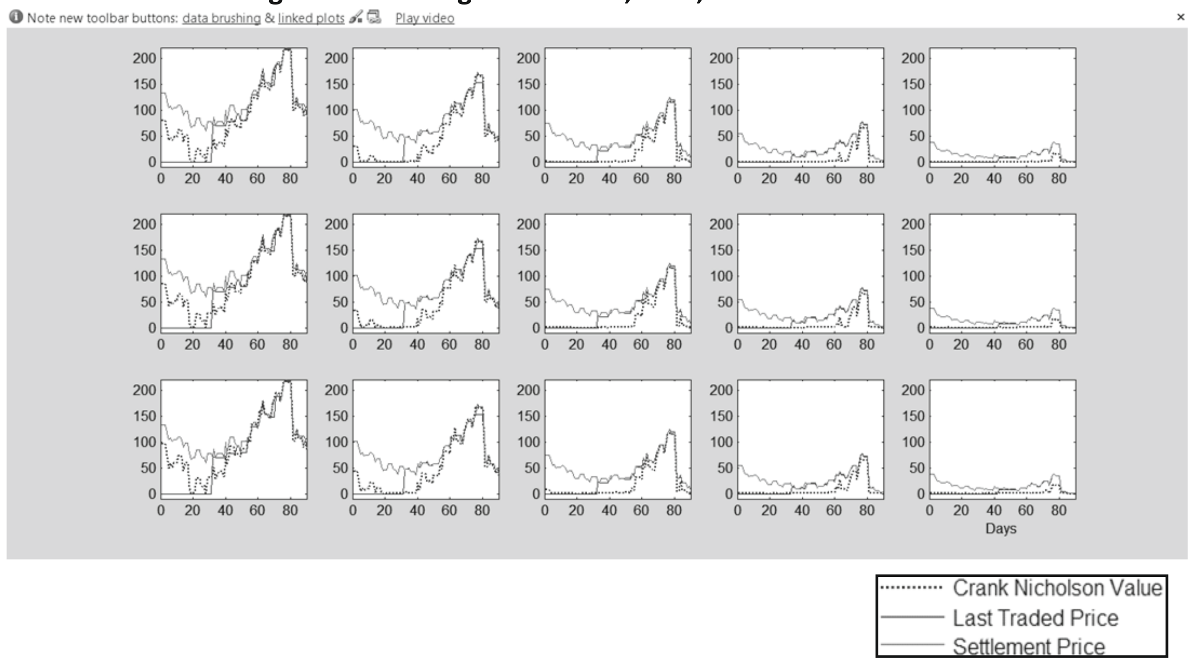
	1200		1250		1300		1350		1400	
	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
Mean	92.76	0.24	48.70	7.67	21.27	31.69	7.61	69.54	1.25	114.70
Std Dev	58.90	1.64	51.38	14.41	35.10	33.05	17.45	48.94	3.82	59.53
Skewness	0.53	8.52	0.95	2.26	1.60	0.77	2.46	0.03	3.42	-0.39
Kurtosis	-0.58	75.98	-0.28	4.69	1.30	-0.55	4.97	-1.14	10.82	-0.82
Minimum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.26
Maximum	217.41	15.00	167.38	65.00	117.35	115.00	67.33	165.00	17.56	215.00

values deviating from normality at the volatility rate of 10%.

The Table 6 depicts the descriptive statistics of five different strike price levels at 30% volatility. When 30% volatility is applied on the CN method to compute the values of call and put contracts, they show the values - (86.32, 88.41); (0.24, 0.24); (43.85, 45.43); (7.78, 7.77); (18.95, 19.62); (32.86, 32.41); (6.22, 6.67); (70.16, 69.98); (0.81, 0.94); (114.78, 114.78), which are more or less same average values. The volatilities are also more or less similar at 10% and 30% (58.91, 58.94); (1.66, 1.65); (51.03, 58.33); (14.78, 14.63); (33.97, 34.41); (33.16, 33.15); (16.74, 16.96); (48.54, 48.68); (3.35, 3.51); (57.68, 51.16). The skewness and kurtosis values are also approximately equal for the pairs of call and put contracts. The call contracts are normal when the contracts are in the money ; the same are skewed to the right when they are out of money. The put contracts are just opposite to call contracts, they are normal when they are out of money, but become skewed to the left when they are in the money.

The Table 7 shows the option descriptive statistics computed at 50% volatility at the same strike prices as earlier computations. The same pattern could be observed when the volatility of returns of underlying is increased to 0.5. Only there is a marginal increase in the average option values when the call and put contracts are in the money and stable when they are out of money. Skewness and kurtosis patterns and signs are similar except a marginal increase.

Figure 6 . Convergence of LTPs, STPs, and CN Call Values



In the Figure 6, the price path created by the CN central difference method is compared with the STPs and LTPs for their convergence at different levels of volatility. The STPs are available from the date of issue of call contracts, while the LTPs are available only from the date of trading. Usually, the trading starts from the 40th day onwards approximately. This shows the cautious behavior of the derivative investors. Only the contracts start trading and the LTPs and STPs exactly merge with each other in all strike prices. The thin dotted line is not visible because they go with the *X* axis during the initial 40 days, and then it goes along with the STP line exactly. The CN values are given in the thick line (without dots). There are three panels in the Figure 6 depicting the convergence of various option values in order of strike prices of ₹1200, ₹ 1250, ₹ 1300, ₹ 1350, and ₹ 1400. The first panel is for a volatility of 0.1, second panel for 0.3, and third panel is for 0.5 volatility. In each row, the first two contracts are in the money, last two contracts are out of money, and the central contract is at the money.

The Figure 6 comprises of three rows of five line graphs for call options. The rows represent 10%, 30%, and 50% volatilities, and the columns represent the five strike price levels. In the Figure 6, the dotted line converges with the LTP and STP lines around the 40th day for the exercise price ₹ 200 contract in all three volatilities. The volatilities do not influence the CN price line significantly ; it seems the CN line is more or less the same. For the ₹1250 contract also, the lines merge with each other after the 55th day. But in the initial days, the dotted line goes below the settlement price ; during the same period, the LTP is zero as there is no trading in this contract during this period. However, the root mean square error (RMSE) decreases from 30 to 20 due to the STP, which is less for the ₹1250 contract than for the ₹1200 contract as the payoff decreases. The convergence takes place at around the 60th day, and the RMSE declines further for the ₹1300 contract for the same reason as stated above. The convergence is poor when the contracts are out of money. The last two graphs in each row show poor convergence. As they are out of money, there is meager trading in these derivative contracts.

The Figure 7 presents the root mean square error of the various option values computed at three different volatilities of 10%, 30%, and 50%. A closer observation of the Figure 7 reveals that the errors (between settlement prices and Crank Nicolson prices) decline steeply when the contracts are arranged from in the money to out of money. In the case of last traded price and Crank Nicolson price, the errors increase initially and later decline steadily at different volatilities. This low error information is useless as the option contacts are out of money.

Figure 7. Call Contracts' Root Mean Squared Error

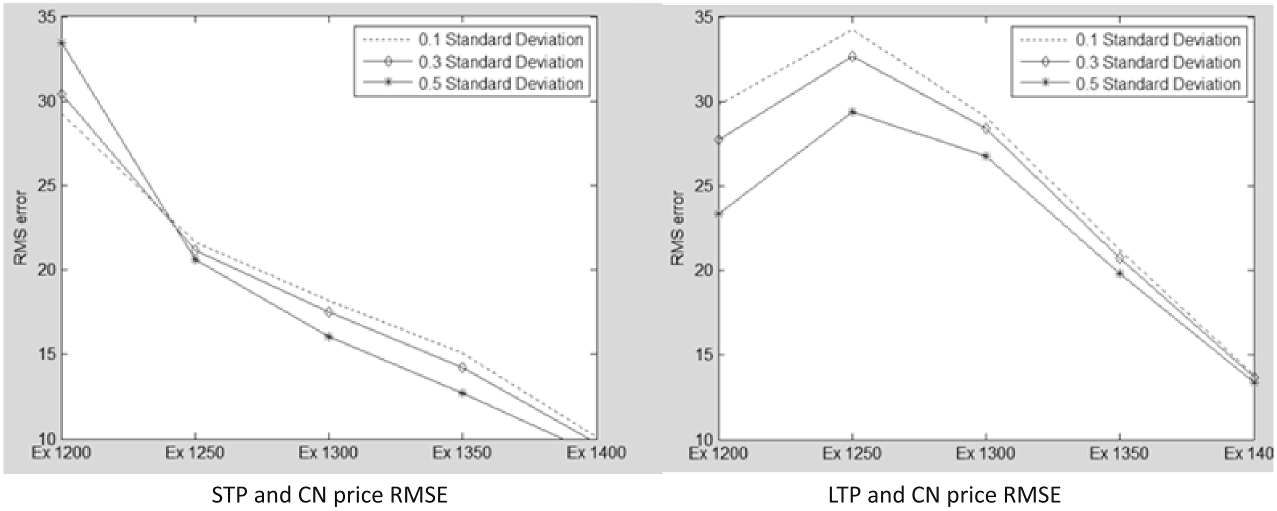
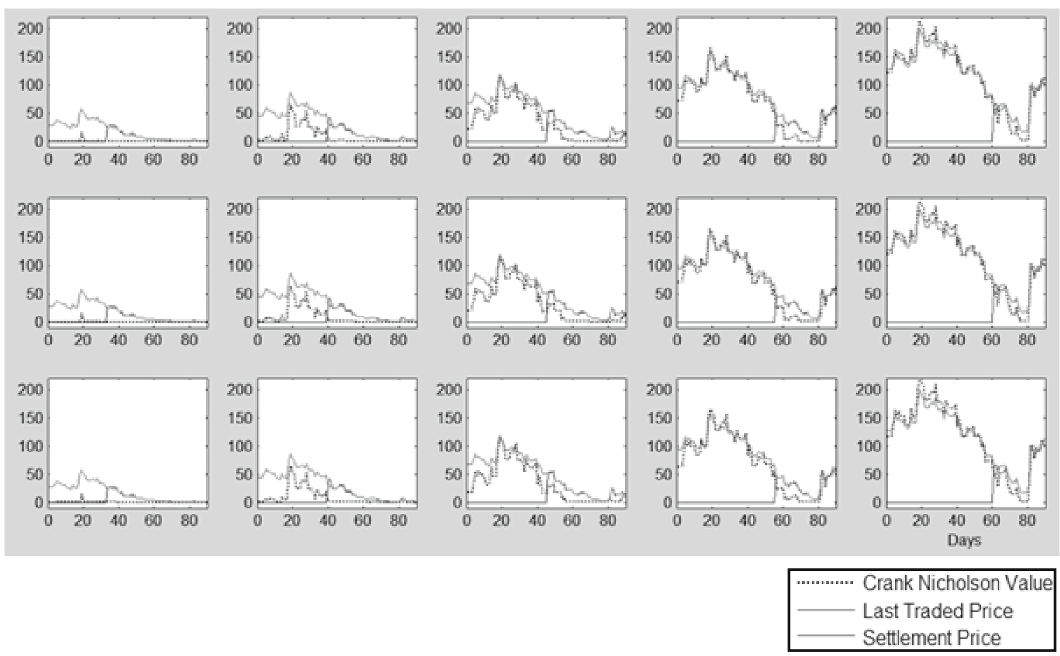


Figure 8. Convergence of LTPs, STPs, and CN Call Values

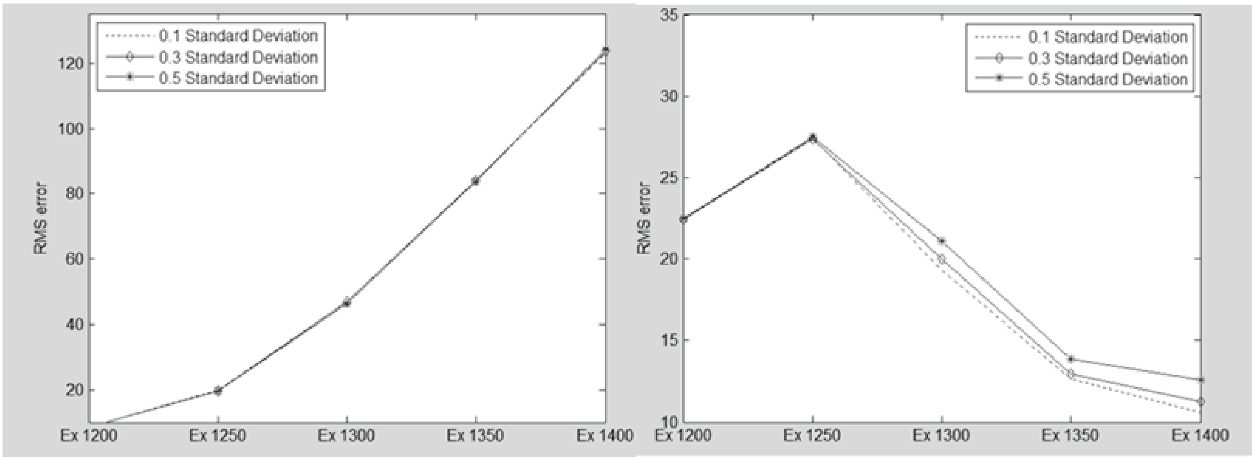


The Figure 8 also presents three rows of graphs at five different strike prices. The first two strike prices are out of money, the final two prices are in the money, and the middle price is at the money. The three rows represent three volatilities of 10%, 30%, and 50% .

The put option contracts are just opposite of call contracts and are in the money when spot prices are less than the exercise price. Here, the spot price on the date of issue is considered as ₹1300 ; as such, the first two contracts are out of money, the last two contracts are in the money, and the central contract is at the money. As in the call option, the CN method fails to converge with STP and LTP in the out of money contracts. The dotted line goes well below the STP.

The Figure 9 is similar to Figure 7 and gives the root mean square error for put option contracts at three different volatilities as usual. In the Figure 9, the RMSE is lesser because the STP line is closer to the *X* axis as the contracts

Figure 9. Put Contracts' Root Mean Squared Error



are out of money. At the money contracts, CN and STP convergence is closer, which produces lesser errors as compared to the out of money contracts, but the LTP and CN produce a larger error as the contracts are traded only for a few days at the end of the life of the contract. In the initial days, since the contract is not traded, there is no price, and therefore, it produces a larger deviation and thus, a larger error. In case of the money contracts, the prices are almost accurately computed by the CN method ; thus, the STP and CN lines converge nicely. However, the LTP and CN methods converge only when the contracts are traded, else the LTPs are zero. Thus, larger deviations arise, and therefore, they show a larger error. In addition, when the contracts are in the money, the CN goes up substantially, and the LTP is zero initially and only during the end the contracts are traded. The volatilities are not sensitive both in call and put contracts in the CN method, and they produce identical price paths.

Implications

An accurate estimation of expected loss in the underlying could be effectively nullified at minimum hedging costs by buying an accurate number of option contracts. Accounting standards prescribe hedge effectiveness as a test to decide whether to apply hedge accounting or not. Hedge accounting will smooth the volatility in earnings. Therefore, the results and the algorithm applied to price options in this article will be highly useful not only for fund managers, but also for companies, regulators, investment analysts, governments, and other stakeholders. As the accounting reporting standards insist on reporting derivative contracts on a balance sheet, the results become more important and pertinent for practical applications.

Conclusion

Computing theoretical prices for option contracts is important as accounting standards specify firms to report derivative contracts at the market price or at fair values on balance sheets and expense the losses immediately in profits. To compute the price path, the CN method of central difference partial difference equation technique was applied. The CN PDF computes the prices for the call and put option contracts closely well to the STPs. The LTPs converge well with the CN prices only when the contracts are in the money. In case of out of money contracts and at the money contracts, the CN method pricing produces poor convergence due to two reasons. The out of money contracts are traded only at the end of their lives at lower prices as their payoff is zero and that too in meagre number. Our findings have practical applications for the industry as they produced a price path for European type call and put contracts more accurately. Regulators and investors can use this algorithm to price or to forecast the option values. In risk management and in hedging decisions, our algorithm will be useful. In addition, the present study also adds to the exiting literature in the option valuation area.

Limitations of the Study and Scope for Further Research

Taking only TCS share price and its option price, and that too only a few contracts in call and put forms, and generalizing the CN method as a good method to assess the effectiveness is questionable. This paper covers only one company - TCS - but the CN algorithm can be used for different companies in different industries with their share price and option price data to assess effectiveness. Application of arbitrary volatilities in estimating option prices is another weakness of this paper.

In the future, researchers can consider the share prices and option prices data of more companies to further validate the results. The arbitrary selection of volatility could be avoided, and instead, implied volatilities can be tried to enhance the results. The share and option price hibernation concept on Saturdays, Sundays, and public holidays could be validated with literature and can be studied with sensitivity analysis in future studies. The celebrated Black - Scholes model, when applied for pricing options, only gives the price at fixed parameter values and does not give a continuous path of prices at every time step. The results are not only for in the money contracts, but they also cover at the money and out of money contracts, thus enhancing the practical use.

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