A Test of Alternative Value-at-Risk Models During Volatile Periods

* *Aparna Prasad Bhat*

Abstract

This paper compared the performance of alternative models for estimating Value at Risk (VaR) of four different currencies against the Indian rupee. I examined whether incorporating a volatility estimate capturing the ARCH effects in the normal linear VaR model yielded a better estimate of market risk than the traditional models based on historical simulation and historical moving average volatility. I tested the effectiveness of different VaR models during the volatile period of June-September 2013 and found that VaR models based on an estimate of time-varying volatility performed better than traditional models during turbulent times.

Keywords: value at risk, ARCH effects, long memory, foreign currency

JEL Classification: G10, G11, G32

Paper Submission Date : February 18, 2015 ; Paper sent back for Revision : May 15, 2015 ; Paper Acceptance Date : July 6 , 2015

The concept of Value-at-Risk (VaR) gained prominence after the Basel Committee on Banking Supervision published an Amendment to the Capital Accord in 1996, which allowed banks to use their own internal risk management models to measure the market risk in their trading books. VaR is a statistical measure which enables banks to capture the market risk of different asset classes and aggregate the same across asset classes. The increasing popularity of VaR among practitioners was accompanied by equal interest from the academic community. While there is abundant literature on the study of VaR for different asset classes and in different countries, the present paper contributes to the existing literature by examining the effectiveness of different VaR models during the recent volatile period of June - September 2013. The Indian rupee experienced severe bouts of volatility after the U. S. Federal Reserve declared its intention in May 2013 of tapering its asset purchases. The rupee depreciated by 11.41% against the U.S. dollar in a space of four months. The depreciation was particularly sharp in the months of June, July, and August 2013, when the rupee lost nearly 15% against the U.S. dollar.

The paper estimates and compares the VaR for four currency pairs, that is, the U.S. dollar-rupee, the pound sterling-rupee, the euro-rupee, and the yen-rupee using the historical simulation approach and the normal linear VaR approach. Under the normal linear VaR approach, I estimate VaR using different estimates of currency volatility such as historical volatility, volatility computed using the exponentially weighted moving average (EWMA), and conditional volatility computed as per the generalized autoregressive conditional heteroskedasticity (GARCH), and the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) models. The question we seek to answer is whether VaR models based on a time-varying volatility perform better than traditional VaR models during a turbulent period.

Literature Review

Early work on VaR include that of Culp, Mensink, and Neves (1998), Hendricks (1996), and Hull and White

* *Assistant Professor,* Department of Finance, K. J. Somaiya Institute of Management Studies and Research, Vidya Vihar, Mumbai - 400 077. E-mail: aparnabhat@somaiya.edu

(1998). The ARCH and GARCH models introduced by Engle (1982) and Bollerslev (1986), respectively and subsequent asymmetric extensions of these models by Rabemananjara and Zakoian (1993) and Glosten, Jagannathan, and Runkle (1993) have been widely used in literature for the estimation of VaR. West and Cho (1995) compared the forecasting performance of GARCH models and other homoscedastic models with respect to five currencies and concluded that while the GARCH models were superior over a one-week horizon, there was no outright winner over the 12 and 24 week horizons. Giot and Laurent (2003) introduced a skewed student APARCH model to model the VaR for portfolios containing both long and short positions.

More recent work included McMillan and Speight (2007), who used EWMA and a variety of GARCH models to estimate the VaR for eight emerging-market stock indices. They concluded that the second- and third-generation GARCH models outperformed the other models. Mokni, Mighri, and Mansouri (2009) studied the effect of sub-prime crisis on VaR estimation using variations of GARCH model with three alternative residual distributions and concluded that the GJR-GARCH model outperformed the GARCH and IGARCH models in both stable and volatile periods.

Fuss, Adams, and Kaiser (2010) used the CAViaR and GARCH-type VaR for commodity futures and found that these models performed better than traditional VaR models. Obi and Sil (2013) estimated the VaR for three international stock indices using the historical and analytical approaches as well as an approach involving time-varying volatility computed using the GARCH model. They found that the VaR computed by the GARCH approach was more robust and reliable than that using the traditional methods. Das, Basu, and Ghoshal (2009) adopted an approach based on a stochastic volatility model (SVM) and Kalman Filter (KF) as against the more traditional EWMA and GARCH-based approach for estimating VaR for Indian security indices. They found that the SVM approach fared better than the EWMA-based technique.

Evaluation of the effectiveness of VaR by means of backtesting has been widely researched. Campbell (2006) and Virdi (2011) reviewed methods and procedures for backtesting VaR. Kupiec (1995) introduced tests for evaluating the effectiveness of VaR.

Data and Methodology

The data set consists of daily exchange rates for USD/INR, EURO/INR, GBP/INR, and JPY/INR obtained from the website of the Reserve Bank of India (RBI) for the in-sample period starting from June 3, 2008 and ending with May 31, 2013, making a total of 1,204 total observations. I chose for out-of-sample analysis the period from June 3, 2013 to September 30, 2013 because this period witnessed high volatility in the rupee. The rupee depreciated nearly 15% against the U.S. dollar during the period from June-August 2013 and then went on to appreciate by nearly 6% during September 2013. I used a rolling window approach for estimating VaR for the out-of-sample period. Thus, observations from day 1 to day T are used to estimate VaR for day $T+1$. Then the window is rolled forward by one day and data from day 2 to day $T+1$ is used to estimate VaR for day $T+2$. For each currency pair, I computed the daily return, r_t as the first logarithmic difference of daily exchange rates, that is, $r_t = \ln(p_t) - \ln(p_{t-1})$. The descriptive statistics for each currency pair are depicted in the Table 1.

All four currency pairs exhibit similar characteristics in that they are negatively skewed and have positive excess kurtosis. The Jarque-Bera statistics are high for all four currency pairs and the hypothesis of normality of the returns distribution is strongly rejected. The Ljung-Box Q-Statistics of order 12 on the squared returns exhibits a very high serial correlation in the second moment. This shows the presence of ARCH effects in the data.

I estimated the one-day VaR at 99% confidence level for each univariate currency return series. The confidence level and holding period are chosen as required by regulators for estimating the risk of a trading book. Furthermore, I estimated the VaR separately for long positions and short positions in each of the four foreign currencies. A long position in the foreign currency would gain from a depreciation of the rupee against that currency, while a short position in the foreign currency would gain from an appreciation of the rupee. The different methods used for computing VaR are explained as follows :

Table 1. Descriptive Statistics for Each Currency Pair

	US Dollar-Rupee	Sterling-Rupee	Euro-Rupee	Yen-Rupee
Mean	0.0002	0.0001	0.0001	0.0003
Median	0.0002	0.0003	0.0004	0.0002
Maximum	0.0249	0.0341	0.0285	0.0399
Minimum	-0.0300	-0.0570	-0.0389	-0.0455
Std.Deviation	0.0059	0.0078	0.0072	0.0103
Skewness	-0.0981	-0.7987	-0.2636	-0.0681
Kurtosis	4.9624	8.1338	5.6775	4.5973
Jarque-Bera	194.9611	1449.051	373.2905	128.8218
Probability	0.0000	0.0000	0.0000	0.0000
*Q*2(12)	198.86	473.27	156.67	668.34

↳ **The Historical Simulation Method :** Under the historical simulation method, the first percentile of the historical returns is the VaR estimate for the long position, and the 99th percentile is the estimate for the short position. For computing the VaR, the positions in the foreign currency need to be converted into domestic currency at the exchange rate prevailing on the day of estimating the VaR. For a long position of 1 million USD, the VaR estimate for June 3, 2013 would be 1.369% of the position value of ₹ 56495800 based on the exchange rate on May 31, 2013. Similarly, the VaR for a short position of 1 million USD would be 1.5088% of ₹ 56495800. The rolling window approach is followed to estimate the VaR for each day in the out-of-sample period.

↳ **The Analytical Approach - Equally Weighted Moving Averages Method :** Under the assumption that the returns are normally distributed, the one-step-ahead VaR for the long position at the 99% confidence level is computed as $z_{1\%}\sigma_t$ while the VaR for the short position is computed as $z_{99\%}\sigma_t$, where $z_{1\%}$ and $z_{99\%}$ are the left and right quantile at 1% and 99%, respectively for the normal distribution and σ_t is the estimated standard deviation for day *t* calculated using the equally-weighted moving average approach as:

$$\sigma_t = \sqrt{\frac{1}{(k-1)} \sum_{i=t-k}^{t-1} (x_i - \mu)^2}$$

where,

σ_t = the estimated standard deviation for day *t,*

k = number of days included in the moving average (estimation window),

x_i = change in asset value on day *I,*

μ = mean change in asset value assumed to be zero as returns are measured over a daily interval.

↳ **The Analytical Approach - Exponentially Weighted Moving Averages (EWMA) Method:** This method attaches different weights to past observations such that the weights decline exponentially and more recent observations are given a higher weight than older observations. The standard deviation for day *t* is computed as :

$$\sigma_t = \sqrt{(1-\lambda) \sum_{i=t-k}^{t-1} \lambda^{t-i-1} (x_i - \mu)^2}$$

The parameter λ is known as the 'decay factor,' which is the rate of decline of weights on the old observations. The size of λ measures the persistence in the influence of older observations in the current estimate of volatility. Instead of following the risk metrics method of using a uniform decay factor of 0.94 for all assets, I have estimated the decay factor separately for each currency pair by maximizing the likelihood function :

$$-ln\sigma_t - \frac{x_t^2}{\sigma_t}$$

Using Excel's solver to maximize the sum of the log likelihood yields the following estimates of λ for each currency pair:

U.S.dollar/rupee : 0.9194 ; sterling-rupee: 0.9075 ; euro-rupee: 0.9600 ; and yen-rupee: 0.9287

Persistence appears to be highest for the euro-rupee pair and lowest for the sterling-rupee pair.

➭ **The Analytical Approach Using Conditional Volatility - GARCH (1,1) Method:** I next consider whether the estimate of VaR can be improved further by using conditional volatility instead of unconditional volatility. The GARCH model is an improvement over the EWMA as it not only captures the persistence in volatility, but also takes into account the mean-reverting nature of volatility. The symmetric GARCH (1,1) model can be expressed in terms of a conditional mean and conditional variance. The equation for the conditional mean is :

$$r_t = \mu + \varepsilon_t$$

where,
μ = expected return at time *t* given the information available at time *t*-1,
ε_t = unexpected return at time *t*,
ε_t = measures the news at time *t* and is positive on arrival of good news and negative on arrival of bad news.

The equation for the conditional variance at time *t* is given by :

$$\sigma^2_t = \omega + \alpha\varepsilon^2_{t-1} + \beta\sigma^2_{t-1}$$

Thus, the conditional variance estimated for time *t* given information available at time *t* -1 is a weighted average of

Table 2. Estimation Results for the Variance Equation Using the GARCH (1,1) Model

Currency Pair	ω	α	β	***LL***	***Q*2(15)**
US Dollar-rupee	0.00000097	0.13966393	0.84011341	4561.8699	12.188551
	[0.000000]	[0.022113]	[0.024185]		(0.6647047)
	(0.001377)	(0.000000)	(0.000000)		
Sterling-rupee	0.00000082	0.05730443	0.92755061	4286.876163	15.00725624
	[0.000000]	[0.009065]	[0.010998]		(0.450894655)
	(0.000511)	(0.000000)	(0.000000)		
Euro-rupee	0.00000037	0.03804568	0.95410274	4316.70301	7.93079825
	[0.000000]	[0.007028]	[0.007394]		(0.92652708)
	(0.012595)	(0.000000)	(0.000000)		
Yen-rupee	0.00000292	0.10246714	0.87071470	3908.90191	16.6620704
	[0.000001]	[0.016872]	[0.019167]		(0.33946384)
	(0.000043)	(0.000000)	(0.000000)		

Standard errors are denoted in square brackets and the *p*-values in round brackets. *LL* refers to the log-likelihood value. *Q*2(15) is the Ljung-Box Q test for serial correlation in the squared standardized residuals with 15 lags. Critical values of Ljung-Box Q statistic are 24.9957 at 5% significance level and 30.5779 at 1% significance level.

three factors: a constant term ω, the impact of unexpected news on day t -1 with a weight α, and the variance forecast for day t -1 with a weight β. The long-term variance can be computed as :

$$\frac{\omega}{(1-\alpha-\beta)}$$

The parameters α, β, and ω are estimated simultaneously by using the maximum likelihood estimation with the restriction that α > = 0, β > = 0, ω> 0, and α + β should be less than 1 in order to ensure stationarity. The error coefficient α measures the reactiveness of the conditional volatility to past shocks and the GARCH coefficient β measures the persistence of past shocks. A large error coefficient and low GARCH coefficient would indicate that current volatility reacts very intensely to past shocks, but the impact of past shocks dies out quickly. When α + β is near unity, it implies that the process has a 'long memory,' which means that the impact of past shocks will persist for several periods. Using the E-views 7 software and assuming a normal distribution, I estimate the parameters of the variance equation for each univariate series separately for the in-sample period from June 3, 2008 to May 31, 2013 and the same are shown in the Table 2.

The error-coefficient is the highest and the GARCH coefficient is the lowest for the dollar-rupee pair, implying that volatility reacts very sharply to news, but the effect of the lagged shocks in the current volatility dies out quickly. The sterling-rupee and euro-rupee pair exhibit low error coefficients and high persistence, implying that volatility reacts slowly to past shocks, but the effect of lagged shocks is felt for a longer time. Having fitted the GARCH model, I checked for the presence of serial correlation in the squared standardized residuals. The Ljung-Box Q-statistic was used to examine the squared standardized residuals from the model to ascertain whether the autocorrelations for the residuals are zero up to the specified lag. I used 15 lags as commonly used in the literature.

The Ljung-Box Q statistic is computed as :

$$Q(m)=n(n+2)\sum_{j=1}^{m}\frac{r_j^2}{n-j}$$

where,
n is the sample size,
r_j^2 is the squared autocorrelation at lag j, and
m is the total number of lags.

Table 3. Estimation Results for the Variance Equation Using the GJR-GARCH (1,1) Model

Currency Pair	ω	α	Λ	β	*LL*	*Q*2(15)
US Dollar-rupee	0.00000094	0.15404539	-0.06007071	0.8538805	4564.222452	13.47017899
	[0.000000]	[0.025542]	[0.026991]	[0.021979]		(0.566030334)
	(0.000549)	(0.000000)	(0.026043)	(0.00000)		
Sterling-rupee	0.00000082	0.05876489	-0.00242587	0.9274530	4286.88519	15.0207268
	[0.000000]	[0.013083]	[0.014431]	[0.01126]		(0.449925248)
	(0.000694)	(0.000007)	(0.866501)	(0.00000)		
Euro-rupee	0.00000042	0.04300884	-0.00813444	0.9525018	4316.835073	7.863634389
	[0.000000]	[0.012021]	[0.013537]	[0.008741]		(0.929133235)
	(0.022654)	(0.000346)	(0.547901)	(0.00000)		
Yen-rupee	0.00000294	0.13437136	-0.07925097	0.8773611	3913.069119	14.71285044
	[0.000001]	[0.024685]	[0.026639]	[0.019275]		(0.472292072)
	(0.000087)	(0.000000)	(0.00293)	(0.00000)		

Standard errors are denoted in square brackets and the p-values in round brackets. *LL* refers to the log-likelihood value. *Q*2(15) is the Ljung-Box Q test for serial correlation in the squared standardized residuals with 15 lags. Critical values of Ljung-Box Q statistic are 24.9957 at the 5% significance and 30.5779 at the 1% significance level.

The null hypothesis of the Ljung-Box test is that there is no autocorrelation among the residuals of the model. The critical value of the test statistic follows a chi-square distribution with degrees of freedom equal to the number of lags. The Q-statistic for the squared standardized residuals at a lag of 15 is less than the critical value (24.9957 at 5% significance and 30.5779 at 1% significance level), and the *p* - value is greater than 0.05 for all the four currency pairs, which is not significant. Therefore, the null hypothesis is not rejected, and it can be concluded that there is no autocorrelation in the second moment and thus, the variance equation is correctly specified. The rolling window approach is used to estimate the conditional variance for each day in the out-of-sample period.

The Analytical Approach Using Conditional Volatility - GJR-GARCH (1,1) Method : It is often observed that the rise in volatility is more pronounced after a large negative return than after a positive return of the same size. This is the so-called 'leverage effect'. In order to capture this asymmetric response of volatility to positive and negative price shocks, Glosten et al. (1993) introduced a variation of the GARCH model, which is known as the GJR-GARCH. The model can be formulated as :

$$\sigma^2_t = \omega + \alpha\varepsilon^2_{t-1} + \lambda 1_{(\varepsilon_{t-1})<0}\varepsilon^2_{t-1} + \beta\sigma^2_{t-1}$$

where,
the indicator function $1_{(\varepsilon_{t-1})<0)}$ takes a value = 1 if $\varepsilon_t < 0$ and zero otherwise. This has the effect of increasing the volatility estimate following a negative return and dampening the volatility estimate after a positive return. The conditional volatility is estimated by maximizing the likelihood function and estimating all parameters α, β, ω, and λ simultaneously. The Table 3 depicts the parameters of the variance equation for each univariate series using the GJR-GARCH(1,1) model for the in-sample period from June 3, 2008 to May 31, 2013.

The error coefficient is again observed to be the highest and the GARCH coefficient lowest for the dollar-rupee pair. It is observed that the λ is negative for all currency pairs. However, it may be noted that each currency pair is quoted as the value of the foreign currency expressed in terms of rupees. If we use the reciprocal of the exchange rate, that is, the value of the rupee in terms of the foreign currency and then estimate the parameters, the λ is positive and the size of the error coefficient changes, but the constant term and the GARCH coefficient are the same, which means that the conditional variance is the same as when we use the exchange rate in terms of the rupee. I tested for serial correlation in the squared standardized residuals up to 15 lags and found that the autocorrelation is not statistically significant, thus implying that the conditional variance equation is correctly specified.

Empirical Results

The one-day VaR estimates according to the different approaches at 99% confidence level for long and short positions individually in each currency pair for the out-of-sample period of June - September 2013 are depicted in the Tables 4, 5, 6, 7, and 8 (Abridged results are given in order to conserve space. Full results can be provided by the author upon request).

Testing the Effectiveness of VaR Estimates : The simplest way of evaluating the effectiveness of a VaR model is by computing its 'hit ratio,' which is simply the ratio of the number of times the VaR estimate was breached to the total number of observations in the out-of-sample period. The VaR limit for a long position on a particular day is breached if the negative return for that day is larger than the VaR estimate. For a short position, the VaR limit is breached if that day's positive return is larger than the VaR estimate. The Table 9 shows the percentage of times in the out-of-sample period when the VaR model failed for each currency position.

The Table 9 shows that the VaR estimates for short positions in each of the four foreign currencies were breached much more frequently during the volatile period of July - September 2013 as compared to the VaR

Table 4. One-Day 99% VaR Under the Historical Simulation Approach

Date	USD 1-day VaR long position	USD 1-day VaR short position	GBP 1-day VaR long position	GBP 1-day VaR short position	Euro 1-day VaR long position	Euro 1-day VaR short position	JPY 1-day VaR long position	JPY 1-day VaR short position
03-Jun-2013	-1.3689%	1.5088%	-2.1865%	1.7192%	-1.8837%	1.9041%	-2.7247%	2.6810%
04-Jun-2013	-1.3689%	1.5088%	-2.1865%	1.7192%	-1.8837%	1.9041%	-2.7247%	2.6810%
05-Jun-2013	-1.3689%	1.5088%	-2.1865%	1.7192%	-1.8837%	1.9041%	-2.7247%	2.6810%
06-Jun-2013	-1.3689%	1.5088%	-2.1865%	1.7192%	-1.8837%	1.9041%	-2.7247%	2.6810%
07-Jun-2013	-1.3689%	1.5088%	-2.1865%	1.7192%	-1.8837%	1.9041%	-2.7247%	2.6810%
10-Jun-2013	-1.3689%	1.5088%	-2.1865%	1.7192%	-1.8837%	1.9041%	-2.7247%	2.6820%
28-Jun-2013	-1.3689%	1.6123%	-2.1865%	1.7393%	-1.8837%	2.0015%	-2.7247%	2.7404%
01-Jul-2013	-1.3852%	1.6123%	-2.1865%	1.7393%	-1.8837%	2.0015%	-2.7247%	2.7404%
02-Jul-2013	-1.3852%	1.6123%	-2.1865%	1.7393%	-1.8837%	2.0015%	-2.7247%	2.7404%
03-Jul-2013	-1.3852%	1.6123%	-2.1865%	1.7393%	-1.8837%	2.0015%	-2.7247%	2.7404%
11-Jul-2013	-1.4210%	1.6123%	-2.1865%	1.7393%	-1.8837%	2.0015%	-2.7247%	2.7404%
12-Jul-2013	-1.4210%	1.6123%	-2.1865%	1.7393%	-1.8837%	2.0015%	-2.7247%	2.7404%
15-Jul-2013	-1.4210%	1.6123%	-2.1865%	1.7393%	-1.8837%	2.0015%	-2.7247%	2.7404%
16-Jul-2013	-1.4210%	1.6123%	-2.1865%	1.7393%	-1.8837%	2.0015%	-2.7247%	2.7404%
17-Jul-2013	-1.4210%	1.6123%	-2.1865%	1.7393%	-1.8837%	2.0015%	-2.7247%	2.7404%
18-Jul-2013	-1.4210%	1.6123%	-2.1865%	1.7393%	-1.8837%	2.0015%	-2.7247%	2.7404%
19-Jul-2013	-1.4210%	1.6123%	-2.1865%	1.7393%	-1.8837%	2.0015%	-2.7247%	2.7404%
22-Jul-2013	-1.4210%	1.6123%	-2.1865%	1.7393%	-1.8837%	2.0015%	-2.7247%	2.7404%
12-Aug-2013	-1.4210%	1.6172%	-2.1865%	1.7393%	-1.8336%	2.0282%	-2.7247%	2.7404%
13-Aug-2013	-1.4210%	1.6172%	-2.1865%	1.7393%	-1.8336%	2.0282%	-2.7247%	2.7404%
14-Aug-2013	-1.4210%	1.6172%	-2.1865%	1.7393%	-1.8336%	2.0282%	-2.7247%	2.7404%
16-Aug-2013	-1.4210%	1.6172%	-2.1865%	1.7393%	-1.8336%	2.0282%	-2.7247%	2.7404%
19-Aug-2013	-1.4210%	1.6172%	-2.1865%	1.7393%	-1.8336%	2.0282%	-2.7247%	2.7404%
20-Aug-2013	-1.4210%	1.6172%	-2.1865%	1.7393%	-1.8336%	2.0282%	-2.7247%	2.7404%
21-Aug-2013	-1.4210%	1.7288%	-2.1865%	1.7451%	-1.8336%	2.1729%	-2.7247%	2.7404%
22-Aug-2013	-1.4210%	1.7288%	-2.1865%	1.7451%	-1.8336%	2.1729%	-2.7247%	2.7404%
23-Aug-2013	-1.4210%	1.7990%	-2.1865%	1.7626%	-1.8336%	2.2539%	-2.7247%	2.7404%
26-Aug-2013	-1.4210%	1.7990%	-2.1865%	1.7626%	-1.8336%	2.2539%	-2.7247%	2.7404%
27-Aug-2013	-1.4210%	1.7990%	-2.1865%	1.7626%	-1.8336%	2.2539%	-2.7247%	2.7404%
28-Aug-2013	-1.4210%	1.8116%	-2.1865%	1.9724%	-1.8336%	2.2539%	-2.7247%	2.7404%
06-Sep-2013	-1.4764%	1.8498%	-2.1865%	1.9900%	-1.8344%	2.2641%	-2.7247%	2.9128%
10-Sep-2013	-1.4764%	1.8498%	-2.1865%	1.9900%	-1.8344%	2.2641%	-2.7247%	2.9128%
11-Sep-2013	-1.4817%	1.8498%	-2.1865%	1.9900%	-1.8344%	2.2641%	-2.7676%	2.9128%
12-Sep-2013	-1.4817%	1.8498%	-2.1865%	1.9900%	-1.8344%	2.2641%	-2.7676%	2.9128%
13-Sep-2013	-1.4817%	1.8498%	-2.1865%	1.9724%	-1.8344%	2.2539%	-2.7676%	2.9128%
16-Sep-2013	-1.4817%	1.8498%	-2.1865%	1.9724%	-1.8344%	2.2539%	-2.7676%	2.7404%
27-Sep-2013	-1.5912%	1.8498%	-2.1865%	1.7451%	-1.8344%	2.1740%	-2.7247%	2.7404%
30-Sep-2013	-1.5912%	1.8498%	-2.1865%	1.7451%	-1.8336%	2.1740%	-2.7247%	2.7404%

Table 5. One-Day 99% VaR Under the Normal Linear VaR Approach Using Equally Weighted Moving Average Volatility Estimates

Date	1-day VaR long position USD	1-day VaR short position USD	1-day VaR long position GBP	1-day VaR short position GBP	1-day VaR long position EUR	1-day VaR short position EUR	1-day VaR long position JPY	1-day VaR short position JPY
03-Jun-2013	-1.3829%	1.3829%	-1.7967%	1.7967%	-1.6548%	1.6548%	-2.3841%	2.3841%
04-Jun-2013	-1.3828%	1.3828%	-1.7968%	1.7968%	-1.6544%	1.6544%	-2.3837%	2.3837%
05-Jun-2013	-1.3824%	1.3824%	-1.7972%	1.7972%	-1.6542%	1.6542%	-2.3838%	2.3838%
06-Jun-2013	-1.3826%	1.3826%	-1.7972%	1.7972%	-1.6534%	1.6534%	-2.3833%	2.3833%
07-Jun-2013	-1.3835%	1.3835%	-1.7983%	1.7983%	-1.6517%	1.6517%	-2.3836%	2.3836%
10-Jun-2013	-1.3836%	1.3836%	-1.7996%	1.7996%	-1.6509%	1.6509%	-2.3879%	2.3879%
28-Jun-2013	-1.4090%	1.4090%	-1.8082%	1.8082%	-1.6680%	1.6680%	-2.4018%	2.4018%
01-Jul-2013	-1.4118%	1.4118%	-1.8121%	1.8121%	-1.6696%	1.6696%	-2.4056%	2.4056%
02-Jul-2013	-1.4133%	1.4133%	-1.8138%	1.8138%	-1.6712%	1.6712%	-2.4082%	2.4082%
03-Jul-2013	-1.4135%	1.4135%	-1.8137%	1.8137%	-1.6718%	1.6718%	-2.4082%	2.4082%
11-Jul-2013	-1.4191%	1.4191%	-1.8133%	1.8133%	-1.6697%	1.6697%	-2.4071%	2.4071%
12-Jul-2013	-1.4202%	1.4202%	-1.8135%	1.8135%	-1.6710%	1.6710%	-2.4072%	2.4072%
15-Jul-2013	-1.4194%	1.4194%	-1.8111%	1.8111%	-1.6698%	1.6698%	-2.4057%	2.4057%
16-Jul-2013	-1.4195%	1.4195%	-1.8111%	1.8111%	-1.6698%	1.6698%	-2.4040%	2.4040%
17-Jul-2013	-1.4210%	1.4210%	-1.8114%	1.8114%	-1.6697%	1.6697%	-2.4047%	2.4047%
18-Jul-2013	-1.4208%	1.4208%	-1.8113%	1.8113%	-1.6698%	1.6698%	-2.4036%	2.4036%
19-Jul-2013	-1.4212%	1.4212%	-1.8123%	1.8123%	-1.6698%	1.6698%	-2.4030%	2.4030%
22-Jul-2013	-1.4212%	1.4212%	-1.8123%	1.8123%	-1.6698%	1.6698%	-2.4031%	2.4031%
12-Aug-2013	-1.4295%	1.4295%	-1.8135%	1.8135%	-1.6589%	1.6589%	-2.4082%	2.4082%
13-Aug-2013	-1.4287%	1.4287%	-1.8140%	1.8140%	-1.6594%	1.6594%	-2.4079%	2.4079%
14-Aug-2013	-1.4297%	1.4297%	-1.8148%	1.8148%	-1.6587%	1.6587%	-2.4045%	2.4045%
16-Aug-2013	-1.4295%	1.4295%	-1.8131%	1.8131%	-1.6587%	1.6587%	-2.4052%	2.4052%
19-Aug-2013	-1.4285%	1.4285%	-1.8150%	1.8150%	-1.6600%	1.6600%	-2.4063%	2.4063%
20-Aug-2013	-1.4277%	1.4277%	-1.8153%	1.8153%	-1.6593%	1.6593%	-2.4056%	2.4056%
21-Aug-2013	-1.4349%	1.4349%	-1.8226%	1.8226%	-1.6668%	1.6668%	-2.4112%	2.4112%
22-Aug-2013	-1.4349%	1.4349%	-1.8228%	1.8228%	-1.6668%	1.6668%	-2.4115%	2.4115%
23-Aug-2013	-1.4487%	1.4487%	-1.8293%	1.8293%	-1.6749%	1.6749%	-2.4164%	2.4164%
26-Aug-2013	-1.4490%	1.4490%	-1.8297%	1.8297%	-1.6759%	1.6759%	-2.4182%	2.4182%
27-Aug-2013	-1.4488%	1.4488%	-1.8296%	1.8296%	-1.6755%	1.6755%	-2.4181%	2.4181%
28-Aug-2013	-1.4561%	1.4561%	-1.8351%	1.8351%	-1.6809%	1.6809%	-2.4243%	2.4243%
05-Sep-2013	-1.4901%	1.4901%	-1.8547%	1.8547%	-1.7128%	1.7128%	-2.4470%	2.4470%
06-Sep-2013	-1.4907%	1.4907%	-1.8564%	1.8564%	-1.7154%	1.7154%	-2.4443%	2.4443%
10-Sep-2013	-1.4904%	1.4904%	-1.8554%	1.8554%	-1.7150%	1.7150%	-2.4440%	2.4440%
11-Sep-2013	-1.5008%	1.5008%	-1.8606%	1.8606%	-1.7180%	1.7180%	-2.4509%	2.4509%
12-Sep-2013	-1.5004%	1.5004%	-1.8587%	1.8587%	-1.7170%	1.7170%	-2.4513%	2.4513%
13-Sep-2013	-1.5005%	1.5005%	-1.8504%	1.8504%	-1.7077%	1.7077%	-2.4486%	2.4486%
16-Sep-2013	-1.4973%	1.4973%	-1.8489%	1.8489%	-1.7069%	1.7069%	-2.4405%	2.4405%
27-Sep-2013	-1.5042%	1.5042%	-1.8330%	1.8330%	-1.6929%	1.6929%	-2.4213%	2.4213%
30-Sep-2013	-1.5049%	1.5049%	-1.8286%	1.8286%	-1.6877%	1.6877%	-2.4213%	2.4213%

Table 6. One-Day 99% VaR Under the Normal Linear VaR Approach Using Exponentially Weighted Moving Average Volatility Estimates

Date	1-day VaR long position USD	1-day VaR short position USD	1-day VaR long position GBP	1-day VaR short position GBP	1-day VaR long position EUR	1-day VaR short position EUR	1-day VaR long position JPY	1-day VaR short position JPY
03-Jun-2013	-1.2045%	1.2045%	-1.1801%	1.1801%	-1.1404%	1.1404%	-2.0139%	2.0139%
04-Jun-2013	-1.1586%	1.1586%	-1.1303%	1.1303%	-1.1177%	1.1177%	-1.9728%	1.9728%
05-Jun-2013	-1.1137%	1.1137%	-1.1622%	1.1622%	-1.1149%	1.1149%	-1.9094%	1.9094%
06-Jun-2013	-1.0980%	1.0980%	-1.1138%	1.1138%	-1.0947%	1.0947%	-1.8519%	1.8519%
07-Jun-2013	-1.1738%	1.1738%	-1.3920%	1.3920%	-1.1488%	1.1488%	-1.9280%	1.9280%
10-Jun-2013	-1.1348%	1.1348%	-1.5050%	1.5050%	-1.2040%	1.2040%	-2.4605%	2.4605%
28-Jun-2013	-2.0238%	2.0238%	-1.6625%	1.6625%	-1.7942%	1.7942%	-2.5510%	2.5510%
01-Jul-2013	-2.1720%	2.1720%	-2.0902%	2.0902%	-1.8487%	1.8487%	-2.7905%	2.7905%
02-Jul-2013	-2.1706%	2.1706%	-2.1651%	2.1651%	-1.8919%	1.8919%	-2.8853%	2.8853%
03-Jul-2013	-2.1022%	2.1022%	-2.0785%	2.0785%	-1.8822%	1.8822%	-2.7812%	2.7812%
11-Jul-2013	-2.1538%	2.1538%	-1.9009%	1.9009%	-1.8044%	1.8044%	-2.5669%	2.5669%
12-Jul-2013	-2.1350%	2.1350%	-1.8750%	1.8750%	-1.8860%	1.8860%	-2.5607%	2.5607%
15-Jul-2013	-2.0663%	2.0663%	-1.8783%	1.8783%	-1.8589%	1.8589%	-2.4755%	2.4755%
16-Jul-2013	-1.9887%	1.9887%	-1.7941%	1.7941%	-1.8235%	1.8235%	-2.3871%	2.3871%
17-Jul-2013	-2.0412%	2.0412%	-1.8249%	1.8249%	-1.8410%	1.8410%	-2.5098%	2.5098%
18-Jul-2013	-1.9575%	1.9575%	-1.7460%	1.7460%	-1.8106%	1.8106%	-2.4306%	2.4306%
19-Jul-2013	-1.9166%	1.9166%	-1.8067%	1.8067%	-1.7795%	1.7795%	-2.3499%	2.3499%
22-Jul-2013	-1.8400%	1.8400%	-1.7553%	1.7553%	-1.7528%	1.7528%	-2.2700%	2.2700%
12-Aug-2013	-1.8736%	1.8736%	-1.9261%	1.9261%	-1.7712%	1.7712%	-2.6151%	2.6151%
13-Aug-2013	-1.8281%	1.8281%	-1.8828%	1.8828%	-1.7595%	1.7595%	-2.5463%	2.5463%
14-Aug-2013	-1.8812%	1.8812%	-1.9074%	1.9074%	-1.7754%	1.7754%	-2.4540%	2.4540%
16-Aug-2013	-1.8059%	1.8059%	-1.8183%	1.8183%	-1.7417%	1.7417%	-2.4223%	2.4223%
19-Aug-2013	-1.7618%	1.7618%	-2.0892%	2.0892%	-1.7694%	1.7694%	-2.4430%	2.4430%
20-Aug-2013	-1.7798%	1.7798%	-2.0748%	2.0748%	-1.7681%	1.7681%	-2.4128%	2.4128%
21-Aug-2013	-2.2417%	2.2417%	-2.6289%	2.6289%	-2.0559%	2.0559%	-2.8300%	2.8300%
22-Aug-2013	-2.1681%	2.1681%	-2.5182%	2.5182%	-2.0146%	2.0146%	-2.7669%	2.7669%
23-Aug-2013	-2.8911%	2.8911%	-3.0481%	3.0481%	-2.3024%	2.3024%	-3.0640%	3.0640%
26-Aug-2013	-2.8702%	2.8702%	-3.0417%	3.0417%	-2.3204%	2.3204%	-3.1963%	3.1963%
27-Aug-2013	-2.7910%	2.7910%	-2.9506%	2.9506%	-2.2823%	2.2823%	-3.0822%	3.0822%
28-Aug-2013	-3.0471%	3.0471%	-3.2019%	3.2019%	-2.4376%	2.4376%	-3.3906%	3.3906%
05-Sep-2013	-3.5914%	3.5914%	-3.5282%	3.5282%	-3.1005%	3.1005%	-4.0316%	4.0316%
06-Sep-2013	-3.5800%	3.5800%	-3.4611%	3.4611%	-3.1088%	3.1088%	-3.9954%	3.9954%
10-Sep-2013	-3.4337%	3.4337%	-3.3000%	3.3000%	-3.0540%	3.0540%	-3.8514%	3.8514%
11-Sep-2013	-3.7378%	3.7378%	-3.4611%	3.4611%	-3.0837%	3.0837%	-4.0926%	4.0926%
12-Sep-2013	-3.5985%	3.5985%	-3.3067%	3.3067%	-3.0367%	3.0367%	-3.9869%	3.9869%
13-Sep-2013	-3.4591%	3.4591%	-3.1547%	3.1547%	-2.9757%	2.9757%	-3.8639%	3.8639%
16-Sep-2013	-3.3191%	3.3191%	-3.0053%	3.0053%	-2.9158%	2.9158%	-3.7299%	3.7299%
27-Sep-2013	-2.9348%	2.9348%	-2.1962%	2.1962%	-2.6216%	2.6216%	-3.0072%	3.0072%
30-Sep-2013	-2.8485%	2.8485%	-2.1299%	2.1299%	-2.5969%	2.5969%	-2.9036%	2.9036%

Table 7. One-Day 99% VaR Under the Normal Linear VaR Approach Using GARCH (1,1) Volatility Estimates

Date	1-day VaR long position USD	1-day VaR short position USD	1-day VaR long position GBP	1-day VaR short position GBP	1-day VaR long position EUR	1-day VaR short position EUR	1-day VaR long position JPY	1-day VaR short position JPY
03-Jun-2013	-1.3490%	1.3490%	-1.2892%	1.2892%	-1.2311%	1.2311%	-2.0643%	2.0643%
04-Jun-2013	-1.2625%	1.2625%	-1.2630%	1.2630%	-1.2114%	1.2114%	-2.0120%	2.0120%
05-Jun-2013	-1.1904%	1.1904%	-1.2800%	1.2800%	-1.2089%	1.2089%	-1.9302%	1.9302%
06-Jun-2013	-1.1659%	1.1659%	-1.2551%	1.2551%	-1.1920%	1.1920%	-1.8604%	1.8604%
07-Jun-2013	-1.2775%	1.2775%	-1.4123%	1.4123%	-1.2399%	1.2399%	-1.9844%	1.9844%
10-Jun-2013	-1.2133%	1.2133%	-1.4821%	1.4821%	-1.2956%	1.2956%	-2.7234%	2.7234%
28-Jun-2013	-2.0169%	2.0169%	-1.6852%	1.6852%	-1.8551%	1.8551%	-2.4316%	2.4316%
01-Jul-2013	-2.2843%	2.2843%	-1.9616%	1.9616%	-1.9017%	1.9017%	-2.8120%	2.8120%
02-Jul-2013	-2.2513%	2.2513%	-2.0090%	2.0090%	-1.9255%	1.9255%	-2.9319%	2.9319%
03-Jul-2013	-2.1095%	2.1095%	-1.9515%	1.9515%	-1.9256%	1.9256%	-2.7640%	2.7640%
11-Jul-2013	-2.1566%	2.1566%	-1.8474%	1.8474%	-1.8199%	1.8199%	-2.4784%	2.4784%
12-Jul-2013	-2.1129%	2.1129%	-1.8483%	1.8483%	-1.9121%	1.9121%	-2.4759%	2.4759%
15-Jul-2013	-1.9802%	1.9802%	-1.8438%	1.8438%	-1.8629%	1.8629%	-2.3569%	2.3569%
16-Jul-2013	-1.8400%	1.8400%	-1.7925%	1.7925%	-1.8241%	1.8241%	-2.2383%	2.2383%
17-Jul-2013	-1.9565%	1.9565%	-1.8110%	1.8110%	-1.8411%	1.8411%	-2.4400%	2.4400%
18-Jul-2013	-1.8057%	1.8057%	-1.7632%	1.7632%	-1.8087%	1.8087%	-2.3316%	2.3316%
19-Jul-2013	-1.7459%	1.7459%	-1.7970%	1.7970%	-1.7770%	1.7770%	-2.2249%	2.2249%
22-Jul-2013	-1.6146%	1.6146%	-1.7671%	1.7671%	-1.7506%	1.7506%	-2.1220%	2.1220%
12-Aug-2013	-1.8269%	1.8269%	-1.8632%	1.8632%	-1.7625%	1.7625%	-2.6380%	2.6380%
13-Aug-2013	-1.7523%	1.7523%	-1.8391%	1.8391%	-1.7489%	1.7489%	-2.5352%	2.5352%
14-Aug-2013	-1.8504%	1.8504%	-1.8524%	1.8524%	-1.7705%	1.7705%	-2.3939%	2.3939%
16-Aug-2013	-0.2251%	0.2251%	-1.8021%	1.8021%	-1.7271%	1.7271%	-2.3545%	2.3545%
19-Aug-2013	-0.5177%	0.5177%	-1.9758%	1.9758%	-1.7593%	1.7593%	-2.3947%	2.3947%
20-Aug-2013	-0.9030%	0.9030%	-1.9699%	1.9699%	-1.7574%	1.7574%	-2.3562%	2.3562%
21-Aug-2013	-2.0443%	2.0443%	-2.3255%	2.3255%	-2.1283%	2.1283%	-2.9672%	2.9672%
22-Aug-2013	-1.9374%	1.9374%	-2.2705%	2.2705%	-2.0734%	2.0734%	-2.8556%	2.8556%
23-Aug-2013	-3.2141%	3.2141%	-2.6451%	2.6451%	-2.4091%	2.4091%	-3.2430%	3.2430%
26-Aug-2013	-3.1183%	3.1183%	-2.6579%	2.6579%	-2.4206%	2.4206%	-3.4001%	3.4001%
27-Aug-2013	-2.9281%	2.9281%	-2.6003%	2.6003%	-2.3606%	2.3606%	-3.1990%	3.1990%
28-Aug-2013	-3.3335%	3.3335%	-2.8144%	2.8144%	-2.5497%	2.5497%	-3.5971%	3.5971%
05-Sep-2013	-3.7024%	3.7024%	-3.0981%	3.0981%	-3.1909%	3.1909%	-4.0441%	4.0441%
06-Sep-2013	-3.6403%	3.6403%	-3.0665%	3.0665%	-3.1850%	3.1850%	-3.9609%	3.9609%
10-Sep-2013	-3.3192%	3.3192%	-2.9815%	2.9815%	-3.1072%	3.1072%	-3.7303%	3.7303%
11-Sep-2013	-3.8925%	3.8925%	-3.0688%	3.0688%	-3.1254%	3.1254%	-4.0521%	4.0521%
12-Sep-2013	-3.5857%	3.5857%	-2.9905%	2.9905%	-3.0546%	3.0546%	-3.8790%	3.8790%
13-Sep-2013	-3.2912%	3.2912%	-2.9093%	2.9093%	-2.9725%	2.9725%	-3.6889%	3.6889%
16-Sep-2013	-3.0097%	3.0097%	-2.8307%	2.8307%	-2.8929%	2.8929%	-3.4893%	3.4893%
27-Sep-2013	-2.3734%	2.3734%	-2.3146%	2.3146%	-2.4805%	2.4805%	-2.5476%	2.5476%
30-Sep-2013	-2.2519%	2.2519%	-2.2678%	2.2678%	-2.4484%	2.4484%	-2.4199%	2.4199%

Table 8. One-Day 99% VaR Under the Normal Linear VaR Approach Using GJR-GARCH (1,1) Volatility Estimates

Date	1-day VaR long position USD	1-day VaR short position USD	1-day VaR long position GBP	1-day VaR short position GBP	1-day VaR long position EUR	1-day VaR short position EUR	1-day VaR long position JPY	1-day VaR short position JPY
03-Jun-2013	-1.4038%	1.4038%	-1.2912%	1.2912%	-1.2589%	1.2589%	-2.1783%	2.1783%
04-Jun-2013	-1.3220%	1.3220%	-1.2605%	1.2605%	-1.2345%	1.2345%	-2.1286%	2.1286%
05-Jun-2013	-1.2525%	1.2525%	-1.2780%	1.2780%	-1.2327%	1.2327%	-2.0472%	2.0472%
06-Jun-2013	-1.2133%	1.2133%	-1.2532%	1.2532%	-1.2149%	1.2149%	-1.9786%	1.9786%
07-Jun-2013	-1.3401%	1.3401%	-1.4151%	1.4151%	-1.2690%	1.2690%	-2.1437%	2.1437%
10-Jun-2013	-1.2730%	1.2730%	-1.4881%	1.4881%	-1.3361%	1.3361%	-3.0425%	3.0425%
28-Jun-2013	-2.1814%	2.1814%	-1.7234%	1.7234%	-1.9957%	1.9957%	-2.6522%	2.6522%
01-Jul-2013	-2.2790%	2.2790%	-1.9717%	1.9717%	-2.0145%	2.0145%	-2.7723%	2.7723%
02-Jul-2013	-2.2102%	2.2102%	-2.0154%	2.0154%	-2.0244%	2.0244%	-2.7812%	2.7812%
03-Jul-2013	-2.0959%	2.0959%	-1.9653%	1.9653%	-2.0147%	2.0147%	-2.6387%	2.6387%
11-Jul-2013	-2.1034%	2.1034%	-1.8564%	1.8564%	-1.8631%	1.8631%	-2.3543%	2.3543%
12-Jul-2013	-2.0371%	2.0371%	-1.8461%	1.8461%	-1.9817%	1.9817%	-2.4214%	2.4214%
15-Jul-2013	-1.9348%	1.9348%	-1.8549%	1.8549%	-1.9486%	1.9486%	-2.3207%	2.3207%
16-Jul-2013	-1.8183%	1.8183%	-1.8022%	1.8022%	-1.9066%	1.9066%	-2.2134%	2.2134%
17-Jul-2013	-1.8563%	1.8563%	-1.8131%	1.8131%	-1.9015%	1.9015%	-2.2768%	2.2768%
18-Jul-2013	-1.7316%	1.7316%	-1.7642%	1.7642%	-1.8683%	1.8683%	-2.1976%	2.1976%
19-Jul-2013	-1.7064%	1.7064%	-1.8090%	1.8090%	-1.8354%	1.8354%	-2.1049%	2.1049%
22-Jul-2013	-1.5956%	1.5956%	-1.7806%	1.7806%	-1.8091%	1.8091%	-2.0237%	2.0237%
13-Aug-2013	-1.8522%	1.8522%	-1.8912%	1.8912%	-1.8599%	1.8599%	-2.7371%	2.7371%
14-Aug-2013	-1.9706%	1.9706%	-1.9101%	1.9101%	-1.8971%	1.8971%	-2.5910%	2.5910%
16-Aug-2013	-1.8461%	1.8461%	-1.8569%	1.8569%	-1.8477%	1.8477%	-2.4975%	2.4975%
19-Aug-2013	-1.7841%	1.7841%	-2.0596%	2.0596%	-1.8958%	1.8958%	-2.5784%	2.5784%
20-Aug-2013	-1.8413%	1.8413%	-2.0569%	2.0569%	-1.9005%	1.9005%	-2.5567%	2.5567%
21-Aug-2013	-2.6415%	2.6415%	-2.4743%	2.4743%	-2.3782%	2.3782%	-3.3342%	3.3342%
22-Aug-2013	-2.4848%	2.4848%	-2.4114%	2.4114%	-2.3137%	2.3137%	-3.1735%	3.1735%
23-Aug-2013	-3.6821%	3.6821%	-2.8542%	2.8542%	-2.7393%	2.7393%	-3.6729%	3.6729%
26-Aug-2013	-3.4966%	3.4966%	-2.8306%	2.8306%	-2.6968%	2.6968%	-3.6058%	3.6058%
27-Aug-2013	-3.2737%	3.2737%	-2.7548%	2.7548%	-2.6191%	2.6191%	-3.3986%	3.3986%
28-Aug-2013	-3.6908%	3.6908%	-3.0146%	3.0146%	-2.8638%	2.8638%	-3.9526%	3.9526%
05-Sep-2013	-3.9416%	3.9416%	-3.4524%	3.4524%	-3.5537%	3.5537%	-4.3829%	4.3829%
06-Sep-2013	-3.7751%	3.7751%	-3.3927%	3.3927%	-3.4887%	3.4887%	-4.1953%	4.1953%
10-Sep-2013	-3.4785%	3.4785%	-3.2994%	3.2994%	-3.3985%	3.3985%	-3.9537%	3.9537%
11-Sep-2013	-3.7086%	3.7086%	-3.3259%	3.3259%	-3.3453%	3.3453%	-3.9879%	3.9879%
12-Sep-2013	-3.4419%	3.4419%	-3.2427%	3.2427%	-3.2596%	3.2596%	-3.7927%	3.7927%
13-Sep-2013	-3.1852%	3.1852%	-3.1561%	3.1561%	-3.1734%	3.1734%	-3.6231%	3.6231%
16-Sep-2013	-2.9429%	2.9429%	-3.0729%	3.0729%	-3.0904%	3.0904%	-3.4255%	3.4255%
27-Sep-2013	-2.2595%	2.2595%	-2.4685%	2.4685%	-2.6029%	2.6029%	-2.4467%	2.4467%
30-Sep-2013	-2.1386%	2.1386%	-2.4123%	2.4123%	-2.5486%	2.5486%	-2.3261%	2.3261%

Table 9. Percentage of Times the VaR Estimate was Breached During the Out-of-Sample Period

Position	Historical Simulation	Normal-Linear Eq WMA	Normal-Linear EWMA	Normal-Linear GARCH (1,1)	Normal-Linear GJR-GARCH (1,1)
USD-Rupee long positions	8.43%	7.23%	0.00%	0.00%	0.00%
USD-Rupee short positions	9.64%	12.05%	7.23%	9.64%	6.02%
Sterling-Rupee long positions	0.00%	2.41%	1.20%	1.20%	1.20%
Sterling-Rupee short positions	7.23%	7.23%	6.02%	6.02%	6.02%
Euro-Rupee long positions	1.20%	1.20%	0.00%	0.00%	0.00%
Euro-Rupee short positions	6.02%	7.23%	7.23%	7.23%	7.23%
Yen-Rupee long positions	1.20%	1.20%	0.00%	0.00%	0.00%
Yen-Rupee short positions	2.41%	8.43%	6.02%	6.02%	6.02%

estimates for long positions. This is because the rupee experienced a sharp decline against all the four currency majors during this period and the severity of the downward move in the rupee was greater than the upward move during September 2013. The Table 9 shows that no single model has outperformed the other models for all the eight currency positions. The normal linear VaR method, based on equally-weighted moving averages shows the worst performance with the highest percentage of breaches for the out-of-sample data. The normal linear VaR based on GJR-GARCH has given the least percentage of breaches for five of the eight positions considered, and hence it emerges as the best model for the sample data under investigation on the basis of the number of breaches only.

I now test whether the percentage of breaches for each VaR model is statistically significant. Back-testing is the process of evaluating the effectiveness of VaR models and is a regulatory requirement. Tests of VaR models can be classified into unconditional coverage tests and tests of independence. Kupiec (1995) introduced the tests of unconditional coverage. The proportion of failures test is one such test, which examines how many times the portfolio VaR is breached over a given period of time. If the number of breaches differs significantly from the level of significance (α*100%) of the sample, then the VaR model is not considered accurate. Thus, if VaR is computed with a confidence level of 95%, the VaR estimate is expected to be breached 5% of the time. This is compared with the actual proportion of violations over the given period. I first computed a likelihood ratio statistic as under :

$$\ln(LR_{uc}) = n_1 \ln(\pi_{exp}) + n_0 \ln(1 - \pi_{exp}) - n_1 \ln(\pi_{obs}) - n_0 \ln(1 - \pi_{obs})$$

where,
n_1 = observed instances of violations,
n_0 = observed instances of non-violations,
π_{exp} = expected proportion of violations, and
π_{obs} = observed proportion of violations.

The Kupiec test statistic is then computed as -2 ln (Lr_{uc}). The asymptotic distribution of the Kupiec unconditional coverage test statistic is chi-squared with one degree of freedom.

Next, I tested the performance of each VaR model for the out-of-sample period using Kupiec's proportion of failures test. The Table 10 shows the computed test statistic for long and short positions in each foreign currency using different VaR models. The test statistic cannot be determined in cases where there is no violation because the formula includes ln (π_{obs}), which cannot be defined if π_{obs} is zero. The out-of-sample period consists of 83 observations. Critical value of the test statistic is 6.634896 at the 1% significance level. I formulated the null hypothesis as follows:

➲ **H_0 :** The number of observed violations is less than or equal to the expected number, and hence, the given VaR model is effective.

Table 10. Unconditional Coverage Test for Out-of-Sample Period

	GARCH(1,1)	GJR-GARCH(1,1)	Historical Simulation	EqWMA	EWMA
USD long position					
Test statistic	Cannot be defined	Cannot be defined	17.98670257	13.72940166	Cannot be defined
Decision	**Cannot be defined**	**Cannot be defined**	**Reject null hypothesis**	**Reject null hypothesis**	**Cannot be defined**
USD short position					
	GARCH(1,1)	**GJR-GARCH(1,1)**	**Historical Simulation**	**EqWMA**	**EWMA**
Test statistic	22.55701416	9.832969447	22.55701416	32.50199213	13.72940166
Decision	**Reject null hypothesis**	**Reject null hypothesis**	**Reject null hypothesis**	**Reject null hypothesis**	**Reject null hypothesis**
GBP long position					
	GARCH(1,1)	**GJR-GARCH(1,1)**	**Historical Simulation**	**EqWMA**	**EWMA**
Test statistic	0.033011109	0.033011109	Cannot be defined	1.194646037	0.033011109
Decision	**Cannot reject null hypothesis**	**Cannot reject null hypothesis**	**Cannot be defined**	**Cannot reject null hypothesis**	**Cannot reject null hypothesis**
GBP short position					
	GARCH(1,1)	**GJR-GARCH(1,1)**	**Historical Simulation**	**EqWMA**	**EWMA**
Test statistic	9.832969447	9.832969447	13.72940166	13.72940166	9.832969447
Decision	**Reject null hypothesis**	**Reject null hypothesis**	**Reject null hypothesis**	**Reject null hypothesis**	**Reject null hypothesis**
EUR long position					
	GARCH(1,1)	**GJR-GARCH(1,1)**	**Historical Simulation**	**EqWMA**	**EWMA**
Test statistic	Cannot be defined	Cannot be defined	0.033011109	0.033011109	Cannot be defined
Decision	**Cannot be defined**	**Cannot be defined**	**Cannot reject null hypothesis**	**Cannot reject null hypothesis**	**Cannot be defined**
EUR short position					
	GARCH(1,1)	**GJR-GARCH(1,1)**	**Historical Simulation**	**EqWMA**	**EWMA**
Test statistic	13.72940166	13.72940166	9.832969447	13.72940166	13.72940166
Decision	**Reject null hypothesis**	**Reject null hypothesis**	**Reject null hypothesis**	**Reject null hypothesis**	**Reject null hypothesis**
JPY long position					
	GARCH(1,1)	**GJR-GARCH(1,1)**	**Historical Simulation**	**EqWMA**	**EWMA**
Test statistic	Cannot be defined	Cannot be defined	0.033011109	0.033011109	Cannot be defined
Decision	**Cannot be defined**	**Cannot be defined**	**Cannot reject null hypothesis**	**Cannot reject null hypothesis**	**Cannot be defined**
JPY short position					
	GARCH(1,1)	**GJR-GARCH(1,1)**	**Historical Simulation**	**EqWMA**	**EWMA**
Test statistic	9.832969447	9.832969447	1.194646037	17.98670257	9.832969447
Decision	**Reject null hypothesis**	**Reject null hypothesis**	**Cannot reject null hypothesis**	**Reject null hypothesis**	**Reject null hypothesis**

I reject the null hypothesis if the test statistic is greater than the critical value and conclude that the given VaR model is not effective. The Table 10 shows the results of the unconditional coverage test. It is evident from the Table 10 that the test-statistic is greater than the critical value in 19 out of 20 cases involving short positions in the foreign currencies, while the results are mixed in case of long positions. Hence, while the results are not conclusive for long currency positions, we can safely reject the null hypothesis for all the VaR models in case of short positions in all currency pairs except the historical simulation model in case of short positions in JPY. Thus, it is clear that when the rupee depreciated sharply against all the four major currencies during the volatile period of July - September 2013, all the VaR models failed to capture the risk of this downward movement of the rupee. None of the VaR models fulfilled the back-testing requirements during the turbulent period of June-September 2013. Therefore, no single VaR model emerges as an accurate model across different currency positions during the sample period.

Conclusion

In this study, I have compared the one-day-ahead estimates of VaR for four currency pairs using two different approaches: the historical simulation approach, which is distribution-free and the normal linear VaR approach, which assumes that asset returns are normally distributed. Within the normal linear VaR approach, I have used different estimates of the standard deviation to estimate the VaR, that is, unconditional volatility estimates based on equally-weighted moving average and exponentially-weighted moving average, and also, conditional volatility estimates based on the symmetric GARCH (1,1) and the asymmetric GJR-GARCH (1,1) models. I have used the period from June to September 2013 for out-of-sample analysis, as the volatility in the foreign exchange market was very high during this period.

The analysis of the number of exceedances of the VaR estimate during the out-of-sample period shows that the normal GJR-GARCH (1,1) model yields better estimates of the Value-at-Risk during turbulent times than the symmetric GARCH model, and the models based on historical volatility and historical simulation. Results from Kupiec's proportion of failures test shows that no single VaR model is accurate across all different currency positions. The present study is most similar to the study undertaken by Obi and Sil (2013), who estimated VaR for three international stock indices using the historical and analytical approaches involving the GARCH model, and the results of the present study are comparable to the results obtained by them. I, too, conclude that judged from the percentage of breaches, the VaR estimates produced by GARCH models can be said to be more robust than those produced by the traditional models.

Implications, Limitations of the Study, and Scope for Further Research

It can be concluded from the study that traditional models of estimating VaR do not succeed in capturing the risk effectively during turbulent periods. VaR models based on conditional volatility forecasts using the GARCH or asymmetric GARCH models may do a better job of estimating portfolio risk during such periods. As the study focuses on a period of recent upheaval in the Indian currency market, it suffers from the limitation of being based on a small sample. There is scope for further research in this area by extending the study to a longer sample period.

References

Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics, 31* (3), 307-327.

Campbell, S.D. (2006). A review of backtesting and backtesting procedures. *Journal of Risk, 9* (2), 1-17.

Culp, C., Mensink, R., & Neves, A. (1998). Value at risk for asset managers. *Derivatives Quarterly, 5* (2), 21-33.

Das, A., Basu, P. N., & Ghoshal, T. K. (2009). Stochastic volatility model for Indian security indices: VaR estimation and backtesting. *Indian Journal of Finance, 3* (9), 43 - 47.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica ,50*(4), 987-1007. DOI: 10.2307/1912773

Fuss, R., Adams, Z., & Kaiser, D. G. (2010). The predictive power of value-at-risk models in commodity futures markets. *Journal of Asset Management, 11* (4), 261-285.

Giot, P., & Laurent S. (2003). Value-at-Risk for long and short trading positions. *Journal of Applied Econometrics, 18* (6), 641-663. DOI: 10.1002/jae.710

Glosten, L. R., Jagannathan R., & Runkle, D. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance, 48* (5), 1779 - 1801. DOI: 10.2307/2329067

Hendricks, D. (1996). Evaluation of value-at-risk models using historical data. *FRBNY Economic Policy Review 2* (1), 39-70.

Hull, J., & White, A. (1998). Incorporating volatility updating into the historical simulation method for value at risk. *Journal of Risk, 1* (1), 5-19.

Kupiec, P. (1995). Techniques for verifying the accuracy of risk management models. *The Journal of Derivatives, 3* (2), 73-84. DOI: 10.3905/jod.1995.407942

McMillan, D. G., & Speight, A. E. H. (2007). Value-at-Risk in emerging equity markets: Comparative evidence for symmetric, asymmetric, and long-Memory GARCH models. *International Review of Finance, 7*(1-2), 1-19. DOI: 10.1111/j.1468-2443.2007.00065.x

Mokni, K., Mighri, Z., & Mansouri, F.(2009). On the effect of subprime crisis on value-at-risk estimation: GARCH family models approach. *International Journal of Economics and Finance, 1* (2), 88-104. DOI: 10.5539/ijef.v1n2p88

Obi, P., & Sil, S. (2013). VaR and time-varying volatility: A comparative study of three international portfolios. *Managerial Finance, 39* (7), 625-640. DOI : http://dx.doi.org/10.1108/03074351311323428

Rabemananjara, R., & Zakoian, J. M. (1993). Threshold ARCH models and asymmetries in volatility. *Journal of Applied Econometrics, 8* (1), 31-49. DOI: 10.1002/jae.3950080104

Reserve Bank of India. (Various Years). *Database on Indian economy : Exchange rate*. Retrieved from http://dbie.rbi.org.in/DBIE/dbie.rbi?site=home

Virdi, N.K. (2011). A review of backtesting methods for evaluating value-at-risk. *International Review of Business Research Papers, 7* (4), 14-24.

West, K. D., & Cho, D. (1995). The predictive ability of several models of exchange rate volatility. *Journal of Econometrics, 69* (2), 367-391.