Modeling S&P CNX Nifty Index Volatility With GARCH Class Volatility Models: Empirical Evidence From India

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INTRODUCTION

Theoretically, basic idealization is that returns follow a stationary time series model with stochastic volatility structure. The presence of stochastic volatility implies that returns are not necessarily independent over time. In the year 1982, Engle proposed a volatility process with time varying conditional variance; the Autoregressive Conditional Heteroskedasticity (ARCH) process. Two important characteristics within financial time series, the fat tails and volatility clustering can be analyzed by the GARCH family models. However, empirical evidence shows that high ARCH order has to be selected in order to catch the dynamics of conditional variance. The high ARCH order implies that many parameters have to be estimated and the calculations get burdensome. The development of autoregressive conditional heteroskedasticity (ARCH) and GARCH models has been nothing short of revolutionary for modeling financial time series, to the extent that Robert Engle, who first introduced the ARCH model, shared the 2003 Nobel Prize in Economics for his discovery. The original GARCH models have spurned a number of extensions, including multivariate versions and applications to option pricing. Bollersley, Chou, and Kroner (1992) present a literature review of some of the important academic studies on ARCH and GARCH modeling in finance. Four years after Engel's introduction of the ARCH process, Bollerslev 1986, proposed the Generalized ARCH (GARCH) model as a natural solution to the problem with the high ARCH orders. This model is based on an infinite ARCH specification and it allows to dramatically reduce the number of estimated parameters from an infinite number to just a few. Financial time series often exhibit some well-known characteristics. First, large changes tend to be followed by large changes and small changes tend to be followed by small changes. Secondly, financial time series often exhibit leptokurtosis, which means that the distribution of their returns is fat-tailed (i.e. relative high probability for extreme values).

The GARCH model successfully captures the first property described above, but fails to capture the fat-tail property of financial data. This has lead to the use of non-normal distributions to a better model of the fat-tailed characteristics. The GARCH model has been so successful in looking at volatilities of single assets that researchers all over the world have been developing various alternatives -multivariate models -for years. But they tend to be so complicated and relatively unreliable that they haven't actually caught on in the same way that univariate GARCH models have. Ever since Bollerslev introduced the GARCH model, new GARCH models have been proposed, e.g. Exponential GARCH (EGARCH), with different characteristics, advantages and drawbacks. In this paper, we capture financial time series characteristics by employing GARCH (p,q) model, and its extensions. There are several alternatives that are widely used. GARCH models explicitly are one of them. With the development of the GARCH model, and the ARCH model, steps have been made to make a scientific way of deciding which methods are working better and which ones are not working as well. ARCH is quite a tongue twister. Fortunately, Robert Engle's Nobel Prize-winning ARCH model is much easier to put into practice than to pronounce, according to the many practitioners who today rely on ARCH, and then subsequently developed the GARCH model, to more accurately predict volatility.

Several studies investigate the performance of GARCH models on explaining volatility of mature stock markets (e.g. Sentana and Wadhwani, 1992; Kim and Kon, 1994; Kearney and Daly, 1998; Floros, 2007; Floros et al., 2007). Mecagni and Sourial (1999) examine the behaviour of stock returns as well as the market efficiency and volatility effects in the Egyptian stock exchange using GARCH models. The results show significant departures from the EMH,

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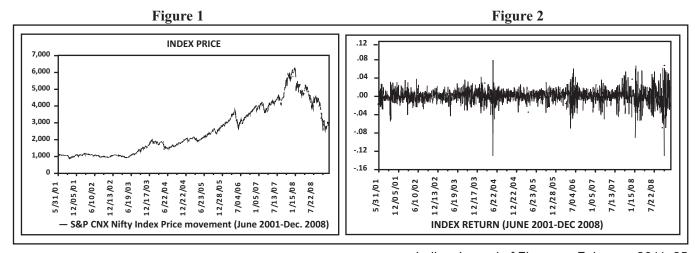
tendency for returns to exhibit volatility clustering and a significant positive link between risk and returns. Furthermore, Tooma (2003) investigates the impact of price limits on volatility dynamics in the Egyptian Stock Exchange using several GARCH models under four different error distributions (Normal, Student-t, GED and Skewedt). The empirical results suggest significant changes in the time varying volatility process. Alberg et al. (2006) estimate stock market volatility of Tel Aviv Stock Exchange indices, for the period 1992-2005, using asymmetric GARCH models. They report that the EGARCH model is the most successful in forecasting the TASE indices. The purpose of this paper is twofold: (i) To explain volatility modeling using recent daily data from emerging Indian stock market, and (ii) To evaluate the performance of the GARCH- family models in explaining financial market risk. We analyzed Nifty stock indices: the barometer of the Indian Economy and Stock market. The motivation for this paper is to add new evidence for Indian stock market modeling of financial time series by explaining volatility clustering. It is not only important to understand the changes of prices of Nifty over time, but also the process by which financing decisions through volatility modeling of Nifty would reached. This paper has been motivated by growing empirical and theoretical evidence that the GARCH effect might be an artifact due to structural changes in the unconditional variance process. The possible causal relation between non-stationarities and the GARCH effect is a recurrent theme in the financial econometric literature (Lamoureux and Lastrapes (1990), Hamilton and Susmel (1994), Cai (1994) among others). In this paper, the researchers investigated volatility forecasting performance of the GARCH(1,1) class models on different time series with and without parameter restrictions.

DATA AND METHODOLOGY

For empirical performance of GARCH class models, we collected daily data of S&P CNX Nifty Index data from www.nseindia.com. The data employed in this study comprises of 1900 daily observations of Nifty Index covering the period from June 1, 2001 to December 31, 2008. S & P CNX Nifty is well-diversified 50 stocks accounting for 23 sectors of the economy. The data are collected for closing prices. The objective behind using the data for the period June 2001 to Dec. 2008 is Index option trading. It was started on NSE from June 4, 2001. This paper finding would be further used for empirical analysis of option pricing.

NATIONAL STOCK EXCHANGE

Established in 1992, the National Stock Exchange (NSE) is the largest financial futures and options exchange in Asia. In 2000, NSE launched the first derivative product in the Indian Stock Market start with Index Futures and subsequently added other products. The F&O segment provides trading in derivative instruments including index futures, index options, stock futures, stock options, interest rate futures. S&P CNX Nifty is owned and managed by India Index Services and Products Ltd. (IISL), which is a joint venture of National Stock Exchange and CRISIL. This study is based on National Stock Exchange Data because NSE accounts for 95 percent of the trading in derivatives in the Indian market.



NIFTY FINANCIAL CHARACTERISTICS

Figure 1 and Figure 2 present the plots of price indices and returns over time. Figure 3 presents the normal distribution histogram of return. Figure 2 describes that the frequency of large and small changes, is rather high which leads us to believe that the data does not come from a normal, but from a heavy tailed (leptokurtic) distribution (relative high probability for extreme values). Large and small values in a log return sample are occurring in clusters. This indicates that there is *dependence in the tails*. This also confirms Mandelbrot quoted (1963): "Large changes tend to be followed by large changes -of either sign- or small changes by small changes". This characteristic is also known as volatility clustering. To confirm that frequency distribution of S&P CNX Nifty Index return does not follow a normal distribution pattern, we plot a graph of Index returns. Figure 3 illustrates this along with QQ plot of index frequency distribution. QQ-plots have been shown to be good tools when deciding what distribution to use. In this paper, the Gaussian (Normal), fat tailed Student-t distribution and the GED are considered.

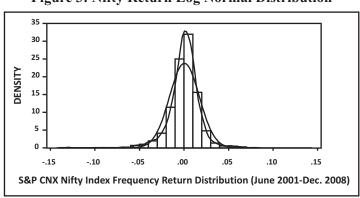


Figure 3: Nifty Return Log Normal Distribution

Figure 3 clearly describes that Index return does not follow log normal distribution pattern as assumed earlier, but a heavy tailed (leptokurtic) distribution along with high peaked (platokurtic).

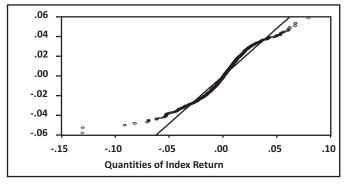


Figure 4: QQ Plot Of Residuals

QQ plot of residuals against normal distribution is also indicating that residuals are significantly departing from

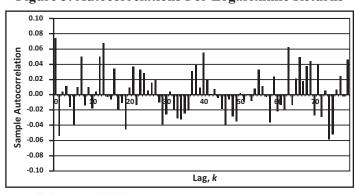


Figure 5: Autocorrelations For Logarithmic Returns

normal distribution pattern having heavy left tail and right tail distribution. The correlogram in Figure 5 clearly indicates the absence of any relationship between the returns on the S&P CNX Nifty index and its lags. These appear to be random, and do not follow any discernable pattern. These results, though encouraging since they indicate that the returns are not correlated, do not imply that the returns are independently distributed. This becomes apparent when the daily logarithmic returns r_i is plotted against time. This plot appears in Figure 2. Figure 2 indicates discernable patterns in the volatility of the returns. In the later part of the graph, the volatility of the index is much larger than in the middle part of the graph. Volatility tends to cluster, and periods of high and low volatility can be identified. The figure clearly indicates that the assumption of constant variance is not consistent with the data as assumed in noble Black-Schole formula used globally for option pricing. This time-varying persistent volatility can be visualized through a correlogram of the return variance. Since the expected daily return is zero, $r\approx 0$ and an estimate of the unconditional variance can be expressed as:

$$\sigma^2 = \frac{1}{n-1} \sum_{t=1}^n r_t^2$$

Any persistence in volatility can, therefore, be identified by plotting the autocorrelations of squared logarithmic returns in a correlogram. This appears in Figure 6. Now the pattern of autocorrelation is much stronger. The autocorrelations are larger and seem to persist for large values of k (no. of lags). This suggests that while the autocorrelations for logarithmic returns presented in Figure 5 are uncorrelated, the variances show strong autocorrelation, and this autocorrelation persists for a long time.

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Figure 6: Autocorrelations Of Squared Logarithmic Returns

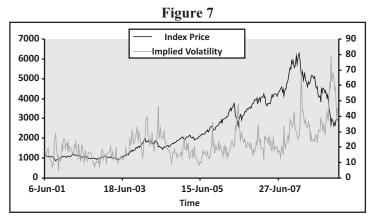
Sample autocorrelations (figure 5) of the Nifty Index returns are small, whereas, the sample autocorrelations of the absolute squared values of Index returns are significantly different from zero even for large lags, accounting for long-range dependence in the data. This behavior suggests that there is some kind of long-range dependence in the data. From Table 2, values of Ljung-Box Q statistics and its p value rejects the null hypothesis of autocorrelation at 1% level

Table 1: Descriptive	Statistics For Da	ly Nifty Stock N	Market Prices And Returns
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Statistics	Index Price	Index Return
Mean	2521.52	0.00049
Median	2043.2	0.001374
Maximum	6287.85	0.079691
Minimum	854.2	-0.13054
Std. Dev.	1430.457	0.016774
Skewness	0.681266	-0.78953
Kurtosis	2.320661	9.391802
Jarque-Bera/ Probability	183.4112/0	3429.955/0
Observations	1899	1899
ADF (Level)/Probability	-1.01983/0.7482	
ADF (Ist Diff)/Prob.	-40.2055/0	
Ljung-Box Q/ Probability		87.388/ 0.012

of significance. While the Jarque-Bera tests reject the null hypothesis of normal distribution for index return. Table 1 gives the descriptive statistics for daily stock market prices and returns. Daily returns are computed as logarithmic price relatives: $r_t = l_n(c_t/c_{t-1})$ where c_t is the daily close price at time t. Furthermore, we present statistics in Table 1 which are calculated using the observations in the full sample (from June 2001 to Dec 2008): Skewness, Kurtosis, Jarque-Bera and Augmented Dickey Fuller (ADF).

Index price series have positive skewness implying that the distribution has a long right tail. On the other hand, the return series have negative skewness implying that the distribution has a long left tail. Changes in stock prices tend to be negatively correlated with changes in volatility, i.e., volatility are higher after negative shocks than after positive shock of same magnitude. This property is called the *leverage effect*. In other words, volatility tends to rise in response to "bad news" (i.e. when an unexpected price drop occurs) and to fall in response to "good news" (i.e. when an unexpected price rise occurs). This feature is called the leverage effect and was first noted by Black (1976), Figure 7 illustrates this effect.



Nifty index level and BS implied volatility each Wednesday during the period 06-06-2001- 31-12-2008. Figure 7 clearly shows the leverage effect, with a correlation in the first difference of the Nifty Index series equaling -0.16992. Earlier, empirical evidence has also shown that financial data is generally not normally distributed, but is instead more skewed and has a greater kurtosis. So, the samples have all financial characteristics: volatility clustering, heavy left tail, leverage effect, long memory and leptokurtosis. The daily returns for index presented in Figure 2 shows that volatility occurs in clusters. Furthermore, in terms of stationarity, the results from the ADF tests providing little evidence that we may reject the null hypothesis of a unit root. The fact that the conditional variance is unobserved has affected the development of volatility models and has made it difficult to evaluate and compare the different models. Some models allow the volatility to react asymmetrically to positive and negative changes in returns.

GARCH MODELS

⊕GARCH

The simplest GARCH model and the most widely used is that which results when q = 1 and p = 1, named GARCH(1,1), $\sigma_{t+1}^2 = \eta + \gamma \varepsilon_t^2 + \lambda \sigma_t^2$

The (1,1) in GARCH (1,1) refers to the presence of a first order autoregressive GARCH term and a first order moving average ARCH term.

M-GARCH

To account for increase in variance results in a higher expected return, GARCH-in-Mean (M-GARCH) models are

$$r_{t} = \phi + \theta \sigma_{t}^{2} + \varepsilon_{t}$$

$$\varepsilon_{t} \approx N(0, \sigma_{t}^{2})$$

$$\sigma_{t+1}^{2} = \eta + \gamma r_{t}^{2} + \lambda \sigma_{t}^{2}$$

also considered; see Engle, Lilien and Robins, 1987. Standard M-GARCH model is given by: If θ is positive (and significant), then increased risk leads to a rise in the mean return, $\theta \sigma_i^2$ can be interpreted as a risk premium.

&E-GARCH

The conditional variance σ_t^2 of r_t gave information at time t, obviously must be non negative with probability one. In GARCH models, this property is assured by making σ_t^2 a linear combination (with positive weights) of positive random variables (as in the GARCH(q,p) case). This formulation leads to the asymmetric GARCH model. The Nelson Exponential-GARCH models were designed to capture the leverage effect noted in Black (1976) and French et al. (1987). A simple variance specification of E-GARCH is given by:

$$\log \sigma_{t+1}^{2} = \eta + \lambda \log \sigma_{t}^{2} + \gamma \left| \frac{\varepsilon_{t}}{\sigma_{t}} \right| + \delta \frac{\varepsilon_{t}}{\sigma_{t}}$$

The logarithmic form of the conditional variance implies that the leverage effect is exponential (so the variance is non-negative). The presence of leverage effects can be tested by the hypothesis that $\delta \not = 0$, then the impact is asymmetric. The GARCH(q,p) model imposes the non-negative constraints on the parameters coefficients, while there is no restrictions on the parameters coefficients in EGARCH model.

®T-GARCH

Furthermore, the Threshold-GARCH (T-GARCH) model was introduced by Zakoian (1994) and Glosten, Jaganathan and Runkle (1993). The T-GARCH specification for the conditional variance is given by:

$$\sigma_{t+1}^{2} = \eta + \sum_{i=1}^{q} \gamma_{i} \varepsilon_{t+1-i}^{2} + \sum_{i=1}^{q} \delta_{i} \varepsilon_{t+1-i}^{2} d_{t+1-i} + \sum_{j=1}^{p} \lambda_{j} \sigma_{t+1-j}^{2}$$

Where $d_{t+l-i} = 1$ if $\varepsilon_{t+l-i} \not = 0$ and $d_{t+l-i} = 0$ otherwise.

From the model, depending on whether ε_{t+l-i} is above or below the threshold value of zero, ε_{t-l-i}^2 can have different effects on the conditional variance σ_{t-l-i}^2 . When ε_{t+l-i} is positive, the (conditional) impact of ε_{t-l-i}^2 on σ_{t-l-i}^2 is ; $\gamma_t \varepsilon_{t-l-i}^2$ when ε_{t+l-i} is negative, the impact is $(\gamma_t + \delta_t) \varepsilon_{t-l-i}^2$. One would expect that δ_t to be positive so that bad news shows that in negative ε_{t+l-i} to have larger impacts on the volatility of the returns. In practice, threshold values different from zero can be used as one would expect that only large shocks attract investors attention. If $\delta \perp 0$, then the leverage effect exists and bad news increases volatility, while if $\delta \neq 0$, the news impact is asymmetric.

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An alternative specification for the conditional volatility process is Component-GARCH. The conditional variance in the CGARCH(1,1) model is given by

$$\sigma_{t+1}^2 = \overline{\eta} + \gamma (\varepsilon_t^2 - \overline{\eta}) + \lambda (\sigma_t^2 - \overline{\eta})$$

The component model shows mean reversion to $\overline{\eta}$ (constant over time), while it allows mean reversion to varying level ψ_t .

$$\sigma_{t+1}^{2} - \psi_{t+1} = \gamma (\varepsilon_{t}^{2} - \psi_{t}) + \lambda (\sigma_{t}^{2} - \psi_{t})$$
$$\psi_{t+1} = \eta + \delta (\psi_{t} - \eta) + \theta_{1} (\varepsilon_{t}^{2} - \sigma_{t}^{2})$$

is the time varying long run volatility.

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An extension of C-GARCH model is Asymmetric component threshold GARCH model. The CT-GARCH model is generated by combining the component model with the asymmetric TGARCH model. This specification introduces asymmetric effects in the transitory equation. The AGARCH model is given by:

$$\begin{split} r_t &= y_t \kappa + \varepsilon_t \\ \psi_{t+1} &= \eta + \delta_1 (\psi_t - \eta) + \theta_1 (\varepsilon_t^2 - \sigma_t^2) + \theta_2 Z_{1t+1} \\ \sigma_{t+1}^2 &- \psi_{t+1} = \gamma (\varepsilon_t^2 - \psi_t) + \delta (\varepsilon_t^2 - \psi_t) d_t + \lambda (\sigma_t^2 - \psi_t) + \theta_3 Z_{2t+1} \end{split}$$

Where Z_{1i} and Z_{2i} are the exogenous variables and d is the dummy variable indicating negative shocks. $\delta \perp 0$ implies transitory leverage effects in the conditional variance.

***P-GARCH**

Finally, Taylor (1986) and Schwert (1989) introduced the standard deviation GARCH model, where the standard deviation is modeled rather than the variance. This model is generalized in Ding *et al.* (1993) with the Power ARCH specification. In the Power-GARCH model, the power parameter β1 of the standard deviation can be estimated rather than imposed, and the optional γ parameters are added to capture asymmetry upto order r:

$$\sigma_{t+1}^{\theta_1} = \eta + \sum_{i=1}^{p} \lambda_j \sigma_{t+1-j}^{\theta_1} + \sum_{t=1}^{q} \gamma_i (\left| \varepsilon_{t+1-i} \right| - \delta_i \varepsilon_{t+1-1})^{\theta_1}$$

Where, $\theta_1 \perp 0$, $|\delta_i| \le 1$ for i=1...r, $\delta_i = 0$ for all i>r, and r\le p. The symmetric model sets $\delta_i = 0$ for all i. Ding, Granger and Engle (1993) proposed the Power ARCH or PARCH model. In its symmetric form it is given by

$$\sigma_{t+1}^{\theta_1} = \eta + \gamma \varepsilon_t^{2\theta_1} + \lambda \sigma_t^{\theta_1}$$

Note that if $\theta_1 = 2$, and $\delta_i = 0$ for all i, the PGARCH model is simply a standard GARCH specifications. Asymmetric effects are presents if $\delta_i \neq 0$.

&I-GARCH

When estimating the parameters in the GARCH model, one often observes that the sum of the parameters is close to one. For the parameter setting:

$$\sum_{i=1}^{q} \gamma_i + \sum_{i=1}^{p} \lambda_j = 1$$

Engle and Bollerslev coined the name *Integrated GARCH* (*IGARCH*). Here, the "integrated" refers to the fact that there might be a unit root problem which could lead to the non-existence of a stationary version of r_i. the IGARCH has a strictly stationary solution, but with infinite variance.

$$\sigma_{t+1}^2 = \sum_{i=1}^q \gamma_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \lambda_j \sigma_{t-j}^2$$

Such that

$$\sum_{i=1}^{q} \gamma_i + \sum_{j=1}^{p} \lambda_j = 1$$

⊗V-GARCH

The V-GARCH, which restricts the constant term to a function of the GARCH parameters and the unconditional variance, where σ^2 is the unconditional variance of the residuals.

$$\eta = \hat{\sigma}^{2} (1 - \sum_{i=1}^{q} \gamma_{i} - \sum_{i=1}^{p} \lambda_{j})$$

EMPIRICAL RESULTS

To analyze the model best fit for Nifty Index financial return series, the empirical result is divided in four broad sub categories: 1) Maximum-likelihood parameters estimations under different distributional assumptions of the innovations are done, 2) diagnostic tests for the standardized residuals for each model, 3) Evaluating best fit model by log likelihood value, 4) model forecasting performance using metrics like RMSE, MAE and MAPE.

Table 2 reports the parameter estimates of all conditional volatility (GARCH(1,1)-family) models defined in the previous section. We conclude that strong GARCH effects are apparent for Nifty Index returns. Also, the coefficient of lagged conditional variance is significantly positive and less than one, indicating that the impact of 'old' news on volatility is significant. The magnitude of the coefficient λ is high for Nifty index, indicating a long memory in the variance as discussed earlier (Figure 6). In addition, the coefficients of the conditional variance in the mean equation of M-GARCH denoted as θ are positive but insignificant for all distributional assumption. This concludes that higher market-wide risk, provided by the conditional variance, will not necessarily lead to higher returns. Furthermore, EGARCH model shows a negative and significant δ parameter for Nifty Index for all distributional assumption. Thus exhibiting the leverage effect in return during the sample period, which is an important feature of the stock return data. Also, the coefficient λ of lagged conditional variance is significantly positive and less than one, indicating that the impact of old news on volatility is significant: the magnitude of the coefficient is high for Nifty Index, indicating a long memory in the variance.

The TGARCH leverage effect term is also significant, and the news impact is asymmetric. For Nifty Index return, the leverage effect exists and bad news increases volatility. In addition, bad news has an impact of 0.23743, 0.218016 and 0.229478 for Gaussian, Student-t and GED distribution. Also, the estimate of δ is greater than the estimate of γ in all three distributional assumptions, which implies that negative shocks have a larger effect on conditional volatility than positive shocks of the same magnitude (figure 7).

The results of the estimation of C-GARCH and CT-GARCH models show mixed findings for Nifty return series. CT-GARCH model shows weak transitory leverage effects in the conditional variances in all three distributional assumptions. However, the estimates of the persistence in the long run component are significant for and close to unity for both C-GARCH and CT-GARCH, except C-GARCH student-t distributional assumption, indicating that the long run component converges very slowly to the steady state.

The results of the estimation of P-GARCH models confirm that the asymmetric effects are present for Nifty index , the asymmetric parameter is positive and significant for all three distributional assumption. For two restricted versions of GARCH models, the sum of ARCH and GARCH coefficients is very close to one in I-GARCH and V-GARCH models, also indicating that volatility shocks are quite persistent as shown earlier for GARCH and M-GARCH. The coefficient of the lagged squared returns is positive and statistically significant for two models under all distributional specifications. It is found that the sum of the estimated parameters in the GARCH model typically increases towards one with increasing sample size. Looking at the estimated parameters for the different distributional assumptions, one sees that the estimated parameter values do not differ much from each other. This indicates that the models are robust. Even if the GARCH family models successfully capture the thick tail returns, and the volatility clustering, they are poor models if we wish to capture the leverage effect since the conditional variance is a function only of the magnitudes of the past values and not their sign.

PARAMETER ESTIMATION IN THE GARCH MODEL

To be able to predict the volatility for a time series, one first has to fit the GARCH-model to the time series in question. This is done via estimation of the parameters in the model. The most common method of this estimation is the *maximum-likelihood estimation (MLE)*.

- * We report the results from GARCH-type models using the method of maximum likelihood, under three different error distributional assumptions.
- **♥** Z-statistics in the parentheses.
- *Significant at the 5% level.

Until this point, it is not obvious which model can capture best the characteristics of Nifty returns. So, the determining point will be by looking at the diagnostic tests for the standardized residuals for each model and compare it with the standardized residuals for the autoregressive for the raw data. The best model will be the model that can reduce the

Table 2: Estimation Result Of GARCH Family Models For Volatility (Variance Specifications)

GARCH (1,1)					Parameters					
Models	Distribution	η	γ	λ	δ	$\theta_{\mathtt{i}}$	$\delta_{\mathtt{1}}$	θ		
	Gaussian	1.03E-05 (7.024027)*	0.166805 (11.89732)*	0.796638 (50.02171)*						
GARCH (1,1)	Student-t	8.04E-06 (3.973987)*	0.150897 (7.15254)*	0.821657 (36.18982)*						
	GED	9.18E-06 (4.48337)*	0.156579 (7.918688)*	0.81021 (36.38978) *						
	Gaussian	1.03E-05 (7.021557)*	0.166928 (11.76986)*	0.796508 (49.85189)*				0.230705 (0.11204)		
M-GARCH (1,1)	Student-t	8.09E-06 (3.97527)*	0.151341 (7.131501)*	0.821047 (35.97154)*				0.719247 (0.38721)		
	GED	9.22E-06 (4.482887)*	0.156977 (7.889308)*	0.809679 (36.20564)*				0.570538 (0.29836)		
	Gaussian	1.56E-06 (8.625077)*	0.037712 (2.013235)	0.772274 (39.58613)*	0.23743 (8.165057)*					
T-GARCH (1,1)	Student-t	0.128E-05 (5.512258) *	0.042399 (1.766026)*	0.788635 (31.74599)*	0.218016 (5.836595)*					
	GED	1.44E-05 (6.052391)*	0.037814 (1.579691) *	0.778788 (30.97934)*	0.229478 (6.155452)*					
	Gaussian	-0.83732 (-11.0161)*	0.289788 (11.46726)*	0.92841 (115.329)*	-0.1338 (-8.72787)*					
E-GARCH (1,1)	Student-t	-0.69568 (-7.131)*	0.277107 (8.201157)*	0.944134 (91.72554)*	-0.12307 (-6.13891)*					
	GED	-0.77265 (-7.65857)*	0.281643 (8.413421)*	0.935584 (87.52474)*	-0.12936 (-6.44763)*					
	Gaussian	5.5E-07 (0.595959)	0.137E-01 (5.995496)*	0.788034 (44.66161)*		2.692632 (7.057848)*				
P-GARCH (1,1) -S	Student-t	1.33E-06 (0.475167)	0.136946 (4.687971)*	0.814069 (33.22777)*		2.417527 (5.024927)*				
	GED	8.57E-07 (0.457057)	0.135276 (4.531952)*	0.802055 (33.13076)*		2.547221 (5.097854)*				
	Gaussian	4.24E-05 (0.969427)	0.137341 (5.822674)*	0.779281 (40.29519)*	0.484694 (5.103767)*	1.768675 (7.476442)*				
P-GARCH (1,1)-A	Student-t	5.76E-05 (0.809495)	0.138458 (4.984159)*	0.799205 (32.70407)*	0.467874 (4.064565)*	1.652499 (5.808959)*				
	GED	5.46E-05 (0.797945)	0.136952 (4.70573)*	0.788133 (31.82639)*	0.488998 (4.015071)*	1.691535 (5.859989)*				
	Gaussian		0.102699 (21.81435)*	0.897301 (190.5964)*						
I-GARCH (1,1)	Student-t		0.102754 (11.54466)*	0.897246 (100.8073)*						
	GED		0.101391 (12.95194)*	0.898609 (114.7908) *						
	Gaussian	1.03E-05	0.166814 (13.44885)*	0.796669 (50.37581)*						
V-GARCH (1,1)	Student-t	8.09E-06	0.149631 (8.268062)*	0.821717 (36.29421) *						
	GED	9.14E-06	0.157504 (9.024302)*	0.810115 (36.44281)*						

	Gaussian	0.00028 (4.986444)*	0.078796 (2.087819)*	0.779958 (10.74808)*	0.976778 (101.9317)*	0.10539 (2.766107)*		
C-GARCH (1,1)	Student-t	0.000295 (2.85336)*	-0.02968 (-0.82986)	0.214873 (0.178593)	0.97074 (67.41048)*	0.15973 (6.519313)*		
	GED	0.000278 (3.686142)*	-0.00981 (-0.49751)	-0.85926 (-2.79203)*	0.966219 (71.50314)*	0.159491 (7.588123)*		
	Gaussian	0.000192 (7.628038)*	-0.10869 (-3.78913)*	0.739631 (18.77285)*	0.257677 (7.397176)*	0.08547 (5.364418)*	0.971465 (136.7135)*	
CT-GARCH (1,1)	Student-t	0.00027 (4.210993)*	-0.30855 (-2.19635)*	1.200198 (8.422292)*	0.951901 (63.80702)*	0.404302 (2.744586)*	0.951901 (63.80702)*	
	GED	0.000186 (5.656681)*	-0.1084 (-2.93742)*	0.760091 (13.62626)*	0.972366 (102.1048)*	0.085683 (4.165678)*	0.972366 (102.1048)*	

kurtosis and shows the highest level of normality (lowest Jarque-Bera value).

Table 3 reports that the T-GARCH, P-GARCH-A and CT-GARCH model have the lowest Jarque-Bera statistic value and reduce the kurtosis values the most under all distributional assumptions. But if we look inside these models closely, its Gaussian distributional assumption under which models are giving low of lowest Jarque-Bera and kurtosis value. Further inspection of models under Gaussian distributional assumption shows that it is T-GARCH model which outperforms other two.

Table 3: Diagnostic Tests For The Standardized Residuals

Model	GARCH(1,1)			M-GARCH(1,1)			T-GARCH(1,1)		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED	Gaussian	Student-t	GED
Mean	-0.053	-0.066	-0.065	-0.053	-0.068	-0.066	-0.015	-0.039	-0.036
Median	0.001	-0.011	-0.010	0.002	-0.013	-0.011	0.033	0.009	0.0128
Maximum	4.001	3.970	3.984	4.003	3.978	3.9910	4.122	4.166	4.128
Minimum	-4.495	-4.606	-4.567	-4.496	-4.608	-4.568	-4.168	-4.337	-4.254
Std. Dev.	0.998	0.999	1.000	0.998	0.999	1.000	1.000	1.000	1.001
Skewness	-0.479	-0.491	-0.485	-0.479	-0.491	-0.485	-0.402	-0.407	-0.403
Kurtosis	4.357	4.410	4.377	4.357	4.409	4.376	4.142	4.199	4.163
Jarque-Bera	218.63	234.02	224.71	218.61	233.60	224.51	154.54	166.51	158.67
Probability	0	0	0	0	0	0	0	0	0
Ljung-Box Q	95.441	95.071	95.018	95.797	96.187	95.887	93.53	92.077	92.161
Probability	0.002	0.003	0.003	0.002	0.002	0.002	0.004	0.005	0.005

Model	E-GARCH(1,1)			P-GARCH(1,1)-A			I-GARCH(1,1)		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED	Gaussian	Student-t	GED
Mean	-0.017	-0.039	-0.037	-0.015	-0.038	-0.035	-0.039	-0.065	-0.062
Median	0.032	0.010	0.013	0.034	0.011	0.014	0.020	-0.004	-0.003
Maximum	4.214	4.193	4.180	4.138	4.177	4.141	4.343	4.262	4.250
Minimum	-4.546	-4.741	-4.638	-4.194	-4.377	-4.290	-6.451	-6.299	-6.311
Std. Dev.	1.000	1.001	1.002	1.000	1.001	1.001	1.083	1.083	1.082
Skewness	-0.434	-0.447	-0.439	-0.405	-0.413	-0.408	-0.615	-0.614	-0.616
Kurtosis	4.290	4.374	4.321	4.154	4.220	4.180	5.178	5.148	5.153
Jarque-Bera	191.474	212.580	199.155	157.258	171.664	162.821	494.985	484.136	486.967
Probability	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ljung-Box Q	92.431	91.075	91.128	93.530	92.044	92.124	98.069	96.342	96.485
Probability	0.005	0.006	0.006	0.004	0.005	0.005	0.001	0.002	0.002

Model	V-GARCH(1,1)				C-GARCH(1,1)			CT-GARCH(1,1)		
	Gaussian	Student-t	GED	Gaussian	Student-t	GED	Gaussian	Student-t	GED	
Mean	-0.053	-0.067	-0.065	-0.054	-0.067	-0.066	-0.058	-0.035	-0.063	
Median	0.002	-0.011	-0.010	0.001	-0.011	-0.010	-0.005	0.014	-0.010	
Maximum	4.001	3.972	3.984	3.777	4.085	4.010	4.057	3.935	4.116	
Minimum	-4.496	-4.622	-4.557	-4.585	-4.724	-4.644	-4.826	-4.217	-4.876	
Std. Dev.	0.999	1.002	0.999	0.999	1.000	1.001	1.020	0.993	1.022	
Skewness	-0.480	-0.492	-0.485	-0.488	-0.488	-0.490	-0.444	-0.435	-0.446	
Kurtosis	4.357	4.409	4.379	4.311	4.439	4.397	4.174	4.182	4.204	
Jarque-Bera	218.669	233.667	224.885	211.249	239.285	230.214	171.579	170.255	177.579	
Probability	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
Ljung-Box Q	95.445	95.049	95.031	95.000	95.506	96.180	90.733	94.453	90.658	
Probability	0.002	0.003	0.003	0.003	0.002	0.002	0.006	0.003	0.006	

In order to examine which one of the three models used fit data best, one must look at the *negative loglikelihood value* at the *maximum point* for the different models. A significant highest likelihood value for a specific distributional assumption in the MLE indicates that this assumption most likely is the best model (better than the other models).

Table 4: Log Likelihood Value

Model		Distribution MLE	
	Gaussian MLE	Student-t MLE	GED MLE
GARCH(1,1)	5414.409	5447.446	5439.497
M-GARCH(1,1)	5414.416	5447.518	5439.539
T-GARCH(1,1)	5440.041	5465.203	5458.506
E-GARCH(1,1)	5429.927	5459.202	5451.609
P-GARCH(1,1) -S	5416.02	5447.952	5440.317
P-GARCH(1,1) -A	5440.37	5465.853	5459.004
I-ARCH(1,1)	5366.299	5422.554	5409.82
V-GARCH(1,1)	5414.409	5447.44	5439.494
C-GARCH(1,1)	5416.117	5447.691	5439.737
CT-GARCH(1,1)	5429.133	5457.755	5449.932

From the Table 4, it is clearly identifiable that T-GARCH AND P-GARCH (asymmetric) outperforms other models. The log likelihood value ranks asymmetric-GARCH first with the highest likelihood value followed by T-GARCH and CT-GARCH. There is close competition in between T-GARCH and P-GARCH models as the likelihood value for these two models are very similar under all distributional assumptions. Generally, it is difficult to draw conclusions by looking at the maximum log-likelihood values. The log-likelihood values do not differ much, which is an implication of that the log-likelihood surface is flat. Larger sample sizes could give us more information, but large samples imply problems with the stationarity assumption.

FORECASTING VOLATILITY OF STOCK MARKET RETURNS

Finally, the forecasting performance of GARCH-Family models is analyzed as the best fit model. We have used data for the period Jan 1, 2008 to Dec 31, 2008 to check the forecasting performance by using the MLE parameters estimates obtained for the period of May 31, 2001 to Dec 31, 2007 under Gaussian distributional assumption for all models. At this point, it is necessary to evaluate the quality of each volatility model estimated. The GARCH family models used to make accurate predictions of the volatility can be measured by the value of the RMSE, MAE and MAPE of Index returns on the volatility forecast.

Table 5: Forecasting Performance

Forecast	Root Mean Squared Error	Mean Absolute Error	Mean Abs. Percent Error	Theil Inequality Coefficient	Bias Proportion	Variance Proportion	Covariance Proportion
GARCH(1,1)	0.028373	0.0211	107.4148	0.95724	0.024314	0.975686	0
M-GARCH(1,1)	0.02854	0.021263	118.4592	0.915692	0.038515	0.870004	0.091481
T-GARCH(1,1)	0.028332	0.021057	103.897	0.964606	0.021515	0.978485	0
E-GARCH(1,1)	0.028329	0.021055	103.6864	0.965168	0.021311	0.978689	0
P-GARCH(1,1)-S	0.028368	0.021094	106.9303	0.958141	0.023959	0.976041	0
P-GARCH(1,1)-A	0.028331	0.021056	103.7907	0.964881	0.021415	0.978585	0
I-GARCH(1,1)	0.028075	0.020942	114.9683	0.952352	0.003532	0.996468	0
V-GARCH(1,1)	0.028373	0.0211	107.4148	0.95724	0.024314	0.975686	0
C-GARCH(1,1)	0.028373	0.0211	107.4434	0.957187	0.024335	0.975665	0
CT-GARCH(1,1)	0.028375	0.021102	107.5938	0.956908	0.024446	0.975554	0

Forecast sample: 1/01/2008 12/31/2008

Included observations: 246

T-GARCH, E-GARCH and P-GARCH-A models outperforms others having lowest value of MAPE among others. Bias proportion is also least for these three models, which indicates that they could be used for modeling conditional volatility forecast for Nifty Index. It is found that the T-GARCH, E-GARCH and P-GARCH models are the best to capture Nifty return characteristics on the basis of log likelihood, residuals skewness, Jarque bera statistics, and forecasting performance. Since it is found that the T-GARCH, E-GARCH and P-GARCH models are the best to capture Nifty return characteristics, the estimated conditional volatility forecast using these models is shown in figure 8. Since the actual volatility is unobserved, we will use the squared return series of each Market Index as a proxy for the realized volatility. A plot of the proxy against the forecasted volatility provides an indication of the ability of GARCH models to track variations in market volatility (see figure 8).

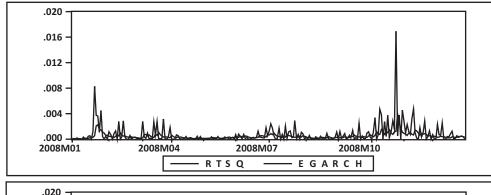
CONCLUSIONS

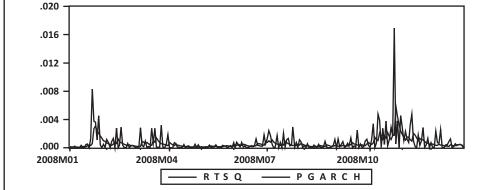
We find strong evidence that Nifty daily returns can be characterized by the GARCH models. The sum of the GARCH coefficients is close to one in almost all cases. That implies persistence of the conditional variance. A large sum of the coefficients in the conditional variance equations implies that a large positive or a large negative return will lead future forecasts of the variance to be high. For Nifty Index, leverage effect has been reported and bad news increases volatility. Finally, the estimates of the persistence in the long run component are significant, indicating that the long run component converges very slowly to the steady state. The correlogram in figure 5 displays a series of autocorrelations that are weak for all lags, and that show no discernable or persistent pattern. Hence, accounting for time-varying volatility in returns through a GARCH process effectively eliminates autocorrelations in squared returns (figure 6). For the estimated parameters under different distributional assumptions, the researchers see that the estimated parameter values do not differ much from each other. This indicates that the models are robust under all

.020 .016 .012 .008 .004 000 2008M07 2008M04 2008M10 2008M01 RTSQ TGARCH

Figure 8: S&P CNX Nifty Index Forecasted Volatility

Figure 8: S&P CNX Nifty Index Forecasted Volatility (Contd.)





distributional assumption. Now, it has been empirically established that using student-t or GED distributional assumption does not make significant difference in terms of coefficient estimates. We conclude that increased risk will not necessarily lead to a rise in the returns. The findings of this paper are strongly recommended to volatility and option traders trading in Nifty Index linked products. Our analysis does not point to a single winner amongst the different volatility models. The result indicates that although not all models outperform, the T-GARCH, E-GARCH and P-GARCH model with normal conditional volatility tends to produce more accurate out-of sample forecasts using standard measures of forecast accuracy but when performance measures in terms of Log likelihood values its T-GARCH and P-GARCH models which outperforms others.

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CONCLUSION

Performance appraisal is the best tool to achieve the employees Performance. Performance appraisal is done to improve performance, not to find a donkey to pin a tail on or blame. The objective of this study was to identify lacunae in the performance appraisal system, which are making performance appraisal system ineffective and by knowing deficient in performance appraisal, one can make an effective performance appraisal system, that would contribute to the goals of the organization & individual improvement because every employee spends a major part of his working life for the organization. In some organizations, mangers spend as much as about 80% of their working life for the organization (including the time they spend at home thinking about ,planning ,discussing etc of the things related to their work). So, it is of paramount importance to develop and utilize the most excellent and effective Performance Appraisal System.

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