

A Model To Predict The BSE Index And Its Volatility

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1. INTRODUCTION

The performance of any stock market – whether it is going up or down is reported in an index, which serves as an important tool for measuring the overall health of the stock market. In most countries, there is more than one index. The BSE index, also called as Sensex compiled in 1986 is India's first index. It is a basket of 30 constituent stocks representing stocks presenting a sample of large, liquid and representative companies. The base year of BSE index is 1978-79 and its base value is 100. The index is widely reported in both domestic and international markets through print as well as electronic media. Due to its wide acceptance amongst the investors, BSE index is regarded to be the pulse of the Indian stock market as it is the language that all investors understand. The data used is the BSE Index reported each day in Times of India and Economics Times, from 21st April, 2004 to 8th June, 2005. In section 1, a model is developed using the US dollar rates and Gold prices with certain lag, which is further augmented by ARIMA process. In section 2, a stochastic trend is fitted, followed by the autocorrelation process. In either of the models, we try to predict not only the future price but also the conditional variances through ARCH process, which gives a good idea about the tranquility or the turbulence of the future.

2. A BSE PREDICTION MODEL WITH US DOLLAR AND GOLD PRICE AS PREDICTORS

The data from 21st April 2004 to 8th June 2005 was obtained for various variables such as currency rates and commodity prices such as gold, silver and oil. Several lag variables were created using these values of data. The Stepwise regression gave the following result, which was the best possible.

The model is,

$$\text{BSE}_t = 1039.255 + 0.94 * \text{BSE}_{(t-1)} + 0.532 * \text{Gold}_{(t-1)} - 0.42 * \text{Gold}_{(t-2)} - 30.343 * \text{USDollar}_{(t-2)} + u_t$$

(527.106) (0.20) (0.128) (0.128) (11.547)

Where BSE_t , Gold_t and USDollar_t are respectively the values of BSE index, Gold and US Dollar at time point t . u_t is the disturbance term.

The ANOVA was highly significant and the R^2 was 0.985 that was the highest possible. The t values for the coefficients of the predictor variable were highly significant. The values shown in the parenthesis are the standard errors of those parameters.

The estimated values of BSE are plotted along with the observed values of BSE in the following graph,

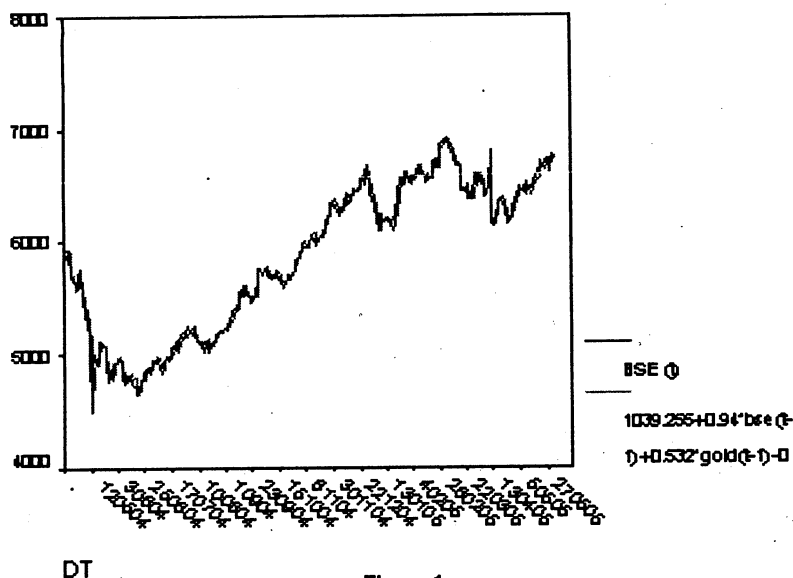


Figure 1.

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The residual term u_t had a mean 1.34 and standard deviation 81.8. ARIMA (2,2) model [Box, Jenkins and Reinsel, (1994)] was fitted to u_t ,

$$u_t = 0.3585 * u_{t-1} - 0.9301 * u_{t-1} + 0.03927 * \varepsilon_{t-1} - 0.8866 * \varepsilon_{t-2} + a_t$$

(0.0781) (0.0811) (0.0917) (0.1041)

where ε_t is white noise and a_t is the residual term.

The ARCH (1) model was fitted to a_t , which gave a $TR^2 = 44.506$. According to Enders (2004) TR^2 converges to χ^2 with 1 d.f. The table value of χ^2 for 1 d.f. and at 5% level of significance is 3.841. Thus we refute the null hypothesis of no ARCH errors. Incidentally ARCH (2) onwards gave significant χ^2 , but the coefficients of $a_{(t-2)}$, $a_{(t-3)}$, ... are approximately zero and they did not contribute to R^2 much. Thus the model obtained was, $a_t = v_t * \sqrt{(4620.646 + 0.398 * a_{(t-1)}^2)}$ where a_t is the residual at time t and v_t is the white noise, which on verification gave $Ev_t \approx 0$ and $Ev_t^2 \approx 1$ and the skewness was -0.037 and kurtosis was -0.965.

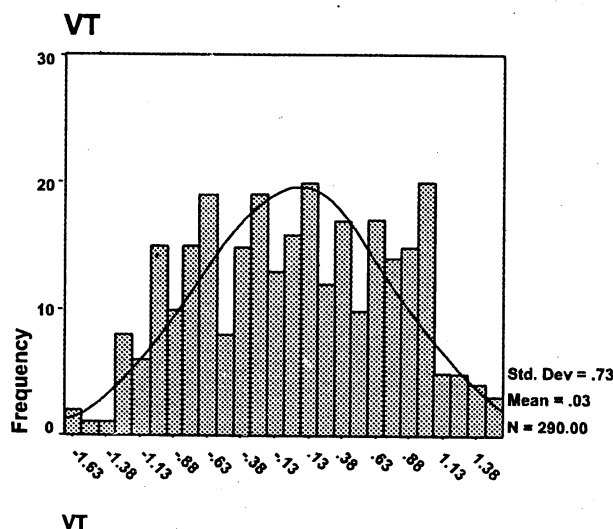


Figure 2.

Now the final model is as below,

$$BSE_t = 1039.255 + 0.94 * BSE_{(t-1)} + 0.532 * Gold_{(t-1)} - 0.42 * Gold_{(t-2)} - 30.343 * USDollar_{(t-2)} + u_t \quad \dots\dots\dots (1)$$

$$\text{Where } u_t = 0.3585 * u_{t-1} - 0.9301 * u_{t-1} + 0.03927 * \varepsilon_{t-1} - 0.8866 * \varepsilon_{t-2} + a_t \quad \dots\dots\dots (2)$$

$$\text{And } a_t = v_t * \sqrt{(4058.478 + 0.393 * a_{(t-1)}^2)} \quad \dots\dots\dots (3)$$

3. A BSE PREDICTION MODEL WITH TREND AND AR (p)-ARCH(q).

In this section we use the same data, but we fit model based on time (t) as an independent variable. It can be seen from figure 1 that a cubic trend can be fitted to the data.

Within the framework of autoregressive time series model, the trend is represented as

$$BSE_t = \mu + \beta t + \rho BSE_{t-1} + e_t$$

Where e_t is the residual term which may contain seasonality and some stationarity random process. The null hypothesis is H_0 : Trend is stochastic i.e. $\rho=1$ against alternative hypothesis that H_1 : Trend is deterministic, i.e. $-1 < \rho < 1$. We do not consider $\rho = -1$, when the BSE time series is non-stationary and restrict ourselves to testing $\rho=1$ because it corresponds to the hypothesis that it is appropriate to transform the time series by differencing [(Dickey-Fuller, 1979, pp.427) and Dickey, Hasza and Fuller (1984)].

The Augmented Dickey-Fuller Tests were conducted [Dickey-Fuller (1981)]. The results are summarized in the following table.

Model	Hypothesis	Test Statistics	Critical Value		Result
			5%	1%	
$\Delta y_t = \gamma y_{t-1} + \sum \beta_i \Delta y_{t-i+1} + \varepsilon_t$	$H_0: \gamma=0$	$\tau=0.422$	-1.95	-2.58	Significant
$\Delta y_t = a_0 + \gamma y_{t-1} + \sum \beta_i \Delta y_{t-i+1} + \varepsilon_t$	$H_0: \gamma=0$	$\tau_\mu = -0.853$	-2.89	-3.51	Significant
	$H_0: a_0 = \gamma=0$	$\Phi_1 = 1.004$	4.71	6.70	Insignificant
$\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \sum \beta_i \Delta y_{t-i+1} + \varepsilon_t$	$H_0: \gamma=0$	$\tau_\tau = -4.834$	-3.45	-4.04	Insignificant
	$H_0: a_2 = \gamma=0$	$\Phi_3 = 12.117$	6.49	8.73	Significant
	$H_0: a_0 =$	$\Phi_2 = 8.177$	4.88	6.50	Significant
	$a_2 = \gamma=0$				

The last model, which includes intercept and trend, does not permit rejection of null hypothesis for unit root and process could be Stochastic.

The trend equation fitted to the data is,

$$BSE_{t+1} = 239.807 + 0.949BSE_t + 0.411 t + v_t$$

(75.914) (0.015) (0.122)

with $R^2 = 0.984$.

The values in the parenthesis are the standard errors of the estimated terms and v_t is the residual term.

The residual values v_t when plotted give the following graph.

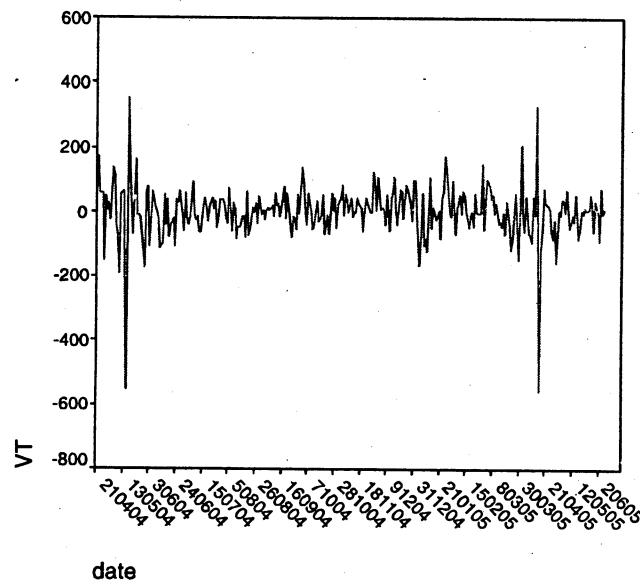


Figure 3.

To the residuals we fitted with AR(4)-ARCH (1) model [Enders (2004), Ghosh & Prajneshu (2003)] which was $v_t = -0.133*v_{t-2} + 0.136*v_{t-4} - 0.0426*\sqrt{h_t} + \xi_t$.

(0.059) (0.059) (0.061)

Where $\xi_t = \varepsilon_t/\sqrt{h_t}$, ε_t follows independent normal distribution with mean 0 and variance 1.

The ARCH factor h_t is estimated as, $h_t = \sqrt{[4038.053 + 0.397*v_{t-1}^2]}$

The $R^2 = 0.043$.

The graph of actual and estimated values of BSE index is shown below.

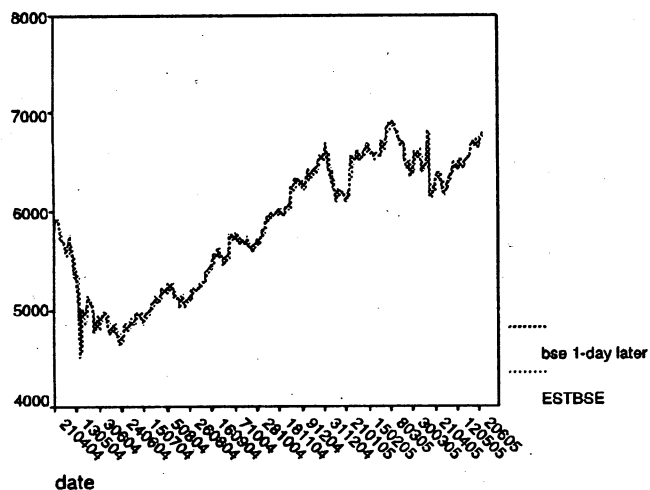


Figure 4.

The residual analysis shows the following histogram with mean 0.05 and S.D. 0.72.

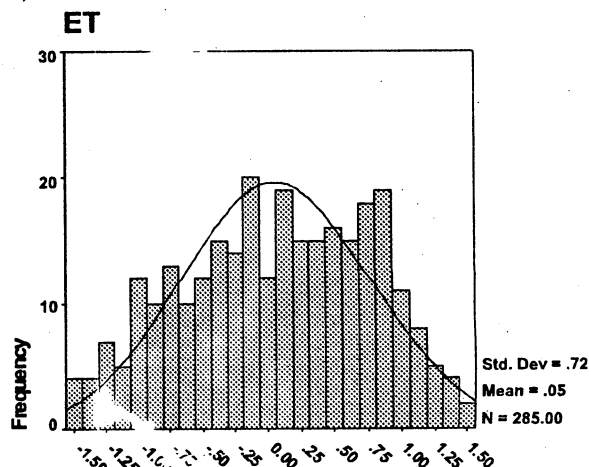


Figure 5.

Thus the final model is as below,

$$BSE_{t+1} = 239.807 + 0.949BSE_t + 0.411t + v_t \quad \text{..... (4)}$$

(75.914) (0.015) (0.122)

$$v_t = -0.133*v_{t-2} + 0.136*v_{t-4} - 0.0426*\sqrt{h_t} + \xi_t \quad \text{..... (5)}$$

(0.059) (0.059) (0.061)

$$\text{Where } \xi_t = \varepsilon_t \sqrt{h_t}, \quad h_t = \sqrt{[4038.053 + 0.397 * v_{t-1}^2]} \quad \text{..... (6)}$$

4. Forecast. In the following table we list the 24 forecasted values of BSE using both the models and the actual values.

Predictions Using Model of Section 2

	Date	BSE	Estimate	ARCH
1	110605	6781.99	6809.91	70.22
2	140605	6832.68	6802.07	70.67
3	150605	6860.18	6807.60	75.64
4	160605	6906.98	6806.04	93.15
5	170605	6900.41	6857.02	73.28
6	180605	6906.52	6914.87	68.18
7	210605	6984.55	6949.97	71.39
8	220605	7076.52	6976.65	92.68
9	230605	7145.34	6995.81	116.27
10	240605	7119.76	7113.52	68.09
11	250605	7148.62	7113.89	71.42
12	280605	7151.08	7179.00	70.22
13	290605	7049.00	7097.23	74.47
14	300605	7119.88	6990.27	106.33
15	10705	7193.85	7071.94	102.64
16	20705	7210.78	7196.15	68.60
17	50705	7277.31	7178.15	92.38
18	60705	7220.25	7173.24	74.16
19	70705	7587.60	7138.27	291.50
20	80705	7145.13	7505.40	237.23
21	90705	7212.08	7152.12	77.79
22	120705	7306.74	7142.14	124.11
23	130705	7303.95	7228.07	83.14
24	140705	7247.99	7238.19	68.26

Predictions Using Model of Section 3

	Date	BSE	Estimate	ARCH
1	110605	6781.99	6832.06	80.25
2	140605	6832.68	6799.57	64.05
3	150605	6860.18	6858.56	65.97
4	160605	6906.98	6858.26	63.97
5	170605	6900.41	6902.31	92.66
6	180605	6906.52	6907.84	76.92
7	210605	6984.55	6917.94	72.14
8	220605	7076.52	6993.66	65.66
9	230605	7145.34	6063.59	85.48
10	240605	7119.76	7129.28	73.56
11	250605	7148.62	7117.89	240.52
12	280605	7151.08	7160.49	279.83
13	290605	7049.00	7152.81	91.21
14	300605	7119.39	7047.81	69.38
15	10705	7193.85	7129.49	68.00
16	20705	7210.78	7172.72	64.23
17	50705	7277.31	7182.14	88.79
18	60705	7220.25	7276.52	84.96
19	70705	7587.60	7117.03	75.15
20	80705	7145.13	7574.96	214.56
21	90705	7212.08	7105.22	177.58
22	120705	7306.74	7258.20	63.82
23	130705	7303.95	7341.43	77.80
24	140705	7247.99	7221.54	69.37

5. CONCLUDING REMARKS

In this paper, efforts are made to develop models for the BSE index which is considered volatile. Though one can see some sudden bursts in the values of index, one can find out that the actual value of BSE lies mostly within predicted value \pm ARCH values (leave alone $\pm 3 \times \text{ARCH}$). One does get an indication of the volatility on a particular day through the values of ARCH.

The models of this paper were built using packages like SPSS-12 and Minitab-13.

6. ACKNOWLEDGEMENT

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7. BIBLIOGRAPHY

- Box, G.E.P., Jenkins, G.M. and Reinsel, G.C. (1994): Time series analysis: Forecasting and control., 3rd edition, Prentice Hall, USA.
- Dickey, D.A. and Fuller, W.A. (1979): Distribution of the estimators for autoregressive time series with a unit root. J. Amer. Statist. Assoc. 74, 427-431.
- Dickey, D.A. and Fuller, W.A. (1981): Likelihood ratio statistics for autoregressive time series with unit root, Econometrica 49, 1057-72.
- Dickey, D.A., Hasza, D.P. and Fuller, W.A. (1984): Testing for unit roots in seasonal time series. J. Amer. Statist. Assoc. 79, 355-367.
- Enders, W. (2004): Applied Econometric Time Series, 2nd Edition, John Wiley & Sons, USA.
- Ghosh, H. and Prajneshu (2003): Non-linear time series modeling of volatile onion price data using AR(p)-ARCH(q)-in-mean., Calcutta Statist. Association Bull., 215-216.