Markowitz and Sharpe's Approaches to Portfolio Construction - A Comparison in Indian Context

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The past decade in many ways has been remarkable for securities market in India. It has grown exponentially as measured in terms of amount raised from the market, number of stock exchanges and other intermediaries, the number of listed stocks, market capitalization, trading volumes and turnover on stock exchanges, and investor population. Despite this growth, the number of investors in equities has declined towards the end of the decade. Surveys have cited various reasons for this trend. One of the reasons cited is that the investment in equities is considered to be risky. Equities can earn good returns on investments as compared to other financial instruments like debentures, preference shares, treasury bills etc. But investors are also well aware of the risk associated with equity shares. All this calls for a well thought out investment in Equities.

Keeping this in mind, the study has been conducted to compare the Ex-ante performance of portfolios constructed using Mean-Variance Model of Markowitz and Single Index Model of Sharpe. Few prominent studies conducted in the area are as follows:

Elton, Edwin J.; Gruber, Martin J. and Padberg, Manferd W. [1977] presented a new method for selecting optimal portfolios when upper bound constraints on investments in individual stocks were present and when the variance-covariance matrix of returns possessed a special structure such as that implied by standard single index model. Extending their previous work, more commonly called as EPG approach to Portfolio optimization, it was shown that upper bounds could be dealt with in a more complex fashion that shares many of the features of ranking procedures of standard single index model.

Solnik and Noetzlin [1982] compared the performance of passive and active strategies for US investors over the period 1970-1980, using the Markowitz portfolio optimization framework. They observed that diversifying internationally reduces risk.

Voros, J. [1986] derived efficient portfolio frontier for cases in which short sales are not allowed. It was shown that when all securities were risky, the efficient frontier consisted of a series of monotonously increasing arcs of convex parabolas in the return variance plane.

Madhusoodanan, T.P. [1996] analyzed the problem of optimum asset allocation among risky investment avenues and tested the performance of the constructed portfolios for the following quarter, half year and a full year. The results were encouraging with an average return significantly higher than the market return for all the three test periods.

Chaudhary, Ashish et al [1998] conducted a study to construct an efficient portfolio using the Sharpe's Single Index model and to observe the performance of the portfolio vis-a-vis the market. It was observed that the portfolio considerably outperformed the market giving a compounded return of 40 % while the market gave a return of -4%.

Chander Shekhar and Garg, M.C. [2001] applied the Markowitz optimization model to generate an efficient combination of securities from amongst a conveniently drawn sample of 30 shares from 'A' Group of securities listed on Bombay Stock Exchange.

Mulvey, John M.; Pauling, William R.and Madey, Ronald E.[2003] observed that a multiperiod portfolio model provides significant advantages over traditional single-period approaches-especially for long-term investors. Such a framework can enhance risk-adjusted performance and help investors evaluate the probability of reaching financial goals by linking asset and liability policies.

RESEARCH METHODOLOGY:

Weekly data was collected on market prices of shares and 30 stock BSE Sensitive Index for a period of 7 years ranging from April 1995 to March 2002. The data source was the website of Bombay Stock Exchange. Three samples were drawn namely sample 1, sample 2 and sample 3 containing 15, 15 and 20 Equity Stocks, respectively, using simple random sampling technique from the universe of Equity stocks listed on Bombay Stock Exchange

Six holding periods of one year (1995-96), two years (1995-97), three years (1995-98), four years (1995-99), five years (1995-2000) and six years (1995-2001) were considered to calculate required inputs, viz. returns and variances-co

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variances-co variances of securities, for constructing portfolios using Makowitz approach. Three levels of expected annual returns viz. 15, 20 and 25 percent were taken to construct the optimal portfolios. Fifty four portfolios were constructed with three samples of securities, six holding periods and three levels of expected return. "Solver" tool of Microsoft Excel 97 was used to find the minimum risk portfolio for any given expected return E*. Mathematically, the problem involves finding the minimum portfolio variance, that is,

Minimize $Var(R_p) = \sum \sum x_i x_j \; i_j$ eqn. I

Where $x_i \rightarrow$ weight of security i in the portfolio

$x_j \rightarrow$ weight of security j in the portfolio

$\sigma_{ij} \rightarrow$ Covariance of returns between securities i and j, when $i \neq j$

$\sigma_{ij} \rightarrow$ Variance of Returns, when i = j

The above minimization is subject to three constraints. The first constraint requires that the desired expected return E* be achieved. This is equivalent to requiring the following equation:

$\sum x_i R_i - E^* = 0$ eqn. II

Where R_i average weekly return on security i

The second constraint requires that the weights of securities in the portfolio sum to unity. This constraint is equivalent to requiring the following equation:

$\sum x_i - 1 = 0$ eqn. III

Third constraint requires that the weights of securities in the portfolio should be non-negative, as short sales have been assumed to be banned. Mathematically, this constraint amounts to following equation:

$x_i, x_j \geq 0$ eqn. IV

The inputs required for solving the model were calculated using the following equations:

Weekly return on security i, $r_i = ((SPc-SPp)+ \text{weekly dividend}) / SP_p$

Where $SP_c \rightarrow$ Closing price of security i in the current week,

$SP_p \rightarrow$ Closing price of security i in the preceding week

Weekly dividend is calculated by dividing the annual dividend by 52.

$R_i = (1/N) * \sum r_i$ eqn. V

Where $N \rightarrow$ total number of weekly security returns

Variance of security i, $\sigma_{ii} = \sigma_i^2 = (1/N)* \sum (r_i-R_i)^2$ eqn. VI

Covariance of security i with security j = $(1/N)* \sum(r_i-R_i)(r_j-R_j)$ eqn. VII

In case of portfolio construction using Sharpe's Single Index Model, too, all the three samples were used to construct optimal portfolios. As Sharpe's model is not as complex as the Markowitz Model, because the data and computational requirement for the model is less as compared to the Markowitz model. A composite sample, consisting of the entire three samples taken together, was also used to construct optimal portfolios. In this case, also, the same six holding periods, as had been used in portfolio construction using Markowitz approach, were considered. So by using four samples of securities and six holding periods, twenty-three portfolios were constructed. This has happened because the portfolio formed on the basis of holding period of 2 years, from sample 3 securities, was dropped as it included just one security in the portfolio. Input values for the model are the security returns, unsystematic variance of security return, beta values of securities and market variance. Optimum portfolio construction involves following steps: -

i) Calculation of excess returns to beta ratio for each stock under review and ranking the securities from highest to lowest based on the ratio. Mathematically,

Excess return to beta ratio = $(R_i - r_f)/ \beta_i$ eqn. VIII

Where $r_f \rightarrow$ average weekly risk-free return

β_i beta value of the security, which is calculated as follows: -

$\beta_i = \{\sum(r_i-R_i)(r_m-R_m)\}/ \sum(r_m-R_m)^2$ eqn. IX

Where $r_m \rightarrow$ weekly market return

R_m Average weekly market return

$R_m =(1/N)* \sum r_m$ eqn. X

Risk free return has been taken to be the yield to maturity of the Treasury bill or Government bond of maturity period equal to the holding period considered.

ii) The optimum portfolio consists of investing in all stocks for which $(r_i - r_f)/ \beta_i$ is greater than a particular cutoff point C*.

To find out cutoff point C*, C_i s, which include i stocks, are calculated as follows -

$$C_i = \frac{\sigma_m^2 \sum\{(r_i - r_f)/\sigma_{ei}^2\}\beta_i}{1 + \sigma_m^2 \sum \beta_i^2/\sigma_{ei}^2} \qquad \text{eqn. XI}$$

Where $\sigma_m 2 \rightarrow$ variance in the market index

$\sigma_{ei}^2 \rightarrow$ variance in the stock's movement that is not associated with the movement of the market index, that is, the unsystematic variance.

$$\sigma_m^2 = \sum(r_m - R_m)^2 \qquad \text{eqn. XII}$$

$$\sigma_{ei}^2 = \sigma_i^2 - \beta_i^2\sigma_m^2 \qquad \text{eqn. XIII}$$

Starting from the top ranked security, the value of C_i is calculated upto a point when excess return to beta ratio, for all the 'i' securities included in the calculation of C_i, is more than the value of C_i and for all other securities, the ratios are less than the C_i values. Securities, included upto that point, constitute optimal portfolio. The percentages to be invested in each security is given by

$$x_i = \frac{Z_i}{\sum Z_i} \qquad \text{eqn. XIV}$$

Where $Z_i = (\beta_i/\sigma_{ei}^2)\{((r_i - r_f)/\beta_i) - C^*\}$ eqn. XV

The constructed portfolios were evaluated using well-established risk adjusted performance measures of Sharpe, Treynor, Jenson and Fama, during one year immediately following their formation.

The reward to variability ratio attempted by Sharpe is referred as the Sharpe Ratio. In fact, this ratio is simply the ratio of the reward; defined as the realized portfolio return Rp in excess of the risk free rates, to the variability of return as measured by the standard deviation of returns (σ_p).

$$RVAR_p = (R_p - r_f)/\sigma_p \qquad \text{eqn. XVI}$$

Where $RVAR_p$ is reward to variability ratio.

R_p is calculated as follows: -

$$R_p = \sum x_i R_i \qquad \text{eqn. XVII}$$

σ_p is calculated as follows: -

$$\sigma_p = (\sum\sum x_i x_j \sigma_{ij})^{1/2} \qquad \text{eqn. XVIII}$$

One sample two-tailed test was used to see whether the mean value of reward to variability ratio is significantly different from zero.

Here, the benchmark is additional return of market over risk free return related with market portfolio's total risk

$$RVAR_m = (R_m - r_f)/\sigma_m \qquad \text{eqn. XIX}$$

Where $RVAR_m$ is the reward to variability of the market.

In case $RVAR_p$ is greater than $RVAR_m$ (benchmark), portfolio's performance is better than market.

According to Treynor's ratio, the additional returns of the portfolio over risk free return is expressed in relation to portfolio's systematic risk measured by Beta (β). This is known as reward to volatility (RVOL) and expressed as

$$RVOL_p = (R_p - r_f)/\beta_p \qquad \text{eqn. XX}$$

Where $RVOL_p \rightarrow$ Reward to volatility of the portfolio

β_p Beta of the Portfolio

It is calculated as follows: -

$$\beta_p = \sum x_i \beta_I \qquad \text{eqn. XXI}$$

Here, additional average returns of market over average risk free return ($R_m - r_f$) is the benchmark. Greater value of the portfolio over the market indicates a superior performance of the fund.

According to Jensen, equilibrium average return is the return of the portfolio by the market with respect to systematic risk of the portfolio. This is a return the portfolio should earn with the given systematic risk,

$$EAR_p = r_f + (r_m - r_f)\beta_p \qquad \text{eqn. XXII}$$

Where $EAR_p \rightarrow$ Equilibrium Average Return of portfolio

Difference between equilibrium average return and average return of the portfolio indicates superior / inferior performance of the fund. This is called as *Alpha* (α)

$$\alpha_p = R_p - EAR_p \qquad \text{eqn. XXIII}$$

If *Alpha* is positive, the portfolio has performed better and if it is negative, it has not shown performance upto the benchmark i.e. the market index.

Eugene Fama has provided an analytical framework that elaborates on the three previously discussed risk adjusted return methods and allows a more detailed breakdown of a fund's performance. Fama has given the following components of the portfolio return: -

a) Risk free return, r_f

b) Compensation for systematic risk, which is known as the risk premium.Mathematically, it is given by following equation-

$$\text{Risk premium} = \beta^*(R_m - r_f)$$

c) To achieve above average returns, sometimes the managers have to forsake some diversification, which will have its cost in terms of additional portfolio risk. An additional component of return is needed to compensate for this additional risk. It is given as follows-

Compensation for inadequate diversification = $(R_m - r_f)^*((\sigma_p / \sigma_m) - \beta)$

d) After deducting all these components, the remaining portfolio return is the net superior return due to selectivity and is given by

Net return due to selectivity = $(R_p - r_f) - (\sigma_p / \sigma_m)(r_m - r_f)$ eqn. XXIV

To compare the performance of the portfolios constructed through Markowitz and Sharpe on return, risk and all the above mentioned risk adjusted measures; a two tailed independent samples t-test was performed on the values of these parameters during one year immediately after their formation.

RESULTS AND DISCUSSIONS

The Sharpe's portfolios gives better average return (0.1755%) than Markowitz's portfolios (0.1361%). However, a larger variation (1.13) in the various portfolio returns was observed in case of Sharpe's model. T-value of the difference between two means is -0.1674 and the difference is not significant at 10 percent level of significance.

On the parameter of risk, Markowitz's model gives better results as we have found the risk in this case was found to be at a lower level of 30.6992 than in case of Sharpe (48.8177). T-value of the difference between two series of variances is -2.7851 and the mean difference is significant even at 1 percent level of significance.

In terms of SI, we have obtained better results. With Sharpe's model, the results were better as the positive value is obtained in this case only. T-value of the difference between two SI means is -0.5825 and it is not significant at 10 percent level of significance. In terms of SI-BMS, also, the results are same with Sharpe's model giving better performance. However, the T -value of the difference between two means (-0.7051) is not significant at 10 percent level of significance.

With Treynor's measure (TM) as the performance evaluation parameter of the portfolios, both the models give negative values. However, lesser negative value was found in case of Sharpe's model. T-value of the difference between two means is -0.2939 and the mean difference is not significant at 10 percent level of significance. When the portfolio performance was evaluated on TM-BMT, the results from Sharpe's approach appear to be better, though negative values are observed in both cases. The T-value of the difference between two means is -0.3546 and the difference is, again, not significant at 10 per cent level of significance.

When comparison is made on Jensen's Measure (Alpha) as portfolio performance evaluation measure, we obtained better results with Sharpe's approach as higher average value of Alpha (0.0008) is obtained in this case. However, the T-value of the difference between two means (-0.3495) is not significant at 10 percent level of significance.

Comparing the two portfolio construction approaches on Fama's measure (FM), the results are, again, in favor of Sharpe's approach as we obtained higher average value of the measure (0.0018) was obtained with it. However, the T-value of the difference between two sample means (-0.6067) is not significant at 10 per cent level of significance.

From the above discussion, it is concluded that Markowitz's model is certainly a better approach in reducing portfolio risk as the portfolios formed using this approach differ significantly in terms of risk from the portfolio formed using Sharpe's approach. In terms of other parameters of portfolio performance, there is no significant difference between the two approaches. Therefore, an aggressive investor can very well use Sharpe's Single Index Model for constructing optimal portfolios as this model is simple and easy to use as compared to Markowitz's mean - variance model.

Table I: Comparison of Performance of Markowitz and Sharpe's Portfolios during One Year following their Construction

Sr. No.	Return		Risk		SI		SI-BMS		TM		TM-BMT		Alpha		FM	
	Markowitz	Sharpe	Markowitz	Sharpe	Markowitz	Sharpe	Markowitz	Sharpe	Markowitz	Sharpe	Markowitz	Sharpe	Markowitz	Sharpe	Markowitz	Sharpe
1	0.19	-0.08	97.75	104.37	-0.0052	-0.0309	0.0307	0.0049	-0.0004	-0.0027	0.0012	-0.0010	0.0015	-0.0012	0.0030	0.0005
2	0.15	0.60	25.20	42.37	-0.0029	0.0667	-0.0300	0.0396	-0.0002	0.0043	-0.0012	0.0032	-0.0011	0.0033	-0.0015	0.0026
3	-0.03	-0.43	25.61	63.61	-0.0369	-0.0736	0.0394	0.0028	-0.0018	-0.0041	0.0014	-0.0009	0.0014	-0.0013	0.0020	0.0002
4	0.36	0.08	27.68	47.57	0.0322	-0.0165	-0.1123	-0.1609	0.0029	-0.0011	-0.0037	-0.0077	-0.0022	-0.0076	-0.0059	-0.0111
5	-0.61	-1.20	16.26	28.68	-0.1953	-0.2562	-0.0464	-0.1073	-0.0186	-0.0320	-0.0115	-0.0250	-0.0049	-0.0107	-0.0019	-0.0057
6	0.32	0.28	15.76	49.74	0.0397	0.0171	0.0763	0.0537	0.0046	0.0023	0.0060	0.0037	0.0021	0.0020	0.0030	0.0038
7	0.39	0.24	15.28	34.73	0.0373	0.0007	0.0732	0.0366	0.0020	0.0000	0.0036	0.0017	0.0027	0.0015	0.0029	0.0022
8	0.52	1.08	11.69	26.67	0.1035	0.1774	0.0763	0.1503	0.0050	0.0097	0.0040	0.0086	0.0028	0.0082	0.0026	0.0078
9	-0.84	0.74	21.49	13.06	-0.2165	0.1611	-0.1402	0.2374	-0.0124	0.0091	-0.0092	0.0122	-0.0075	0.0079	-0.0065	0.0086
10	0.32	0.30	25.97	28.99	0.0260	0.0199	-0.1185	-0.1246	0.0018	0.0023	-0.0047	-0.0043	-0.0034	-0.0020	-0.0060	-0.0067
11	-0.59	-1.61	49.83	124.60	-0.1082	-0.1599	0.0407	-0.0110	-0.0167	-0.0477	-0.0097	-0.0407	-0.0044	-0.0152	0.0029	-0.0012
12	0.26	-0.17	7.68	17.31	0.0335	-0.0812	0.0701	-0.0447	0.0027	-0.0170	0.0041	-0.0156	0.0014	-0.0031	0.0019	-0.0019
13	-0.57	-1.71	36.02	81.46	-0.1342	-0.2164	-0.0983	-0.1805	-0.0100	-0.0192	-0.0084	-0.0175	-0.0067	-0.0178	-0.0059	-0.0163
14	0.52	2.61	23.84	46.34	0.0718	0.3605	0.0446	0.4369	0.0054	0.0341	0.0044	0.0372	0.0028	0.0268	0.0022	0.0297
15	0.73	1.81	17.04	65.16	0.1379	0.2011	0.2143	0.0566	0.0072	0.0162	0.0103	0.0097	0.0082	0.0097	0.0088	0.0046
16	0.84	-0.69	33.56	75.52	0.1121	-0.0989	-0.0324	0.0499	0.0094	-0.0063	0.0029	0.0008	0.0020	0.0011	-0.0019	0.0043
17	-0.06	0.70	14.20	50.24	-0.0614	0.0759	0.0875	0.1124	-0.0045	0.0098	0.0025	0.0112	0.0013	0.0062	0.0033	0.0080
18	-0.08	-0.72	11.21	46.44	-0.0725	-0.1402	-0.0360	-0.1043	-0.0217	-0.0101	-0.0203	-0.0085	-0.0023	-0.0080	-0.0012	-0.0071
19	0.19	0.77	99.35	29.83	-0.0050	0.1101	0.0309	0.0829	-0.0004	0.0060	0.0012	0.0050	0.0015	0.0050	0.0031	0.0045
20	0.18	0.83	25.44	21.87	0.0022	0.1427	-0.0249	0.2191	0.0001	0.0071	-0.0009	0.0103	-0.0008	0.0097	-0.0013	0.0102
21	-0.04	1.20	27.15	39.72	-0.0378	0.1608	0.0386	0.0163	-0.0019	0.0142	0.0013	0.0076	0.0014	0.0054	0.0020	0.0010
22	0.35	-1.18	29.06	45.19	0.0299	-0.2016	-0.1145	-0.0528	0.0026	-0.0137	-0.0039	-0.0067	-0.0024	-0.0066	-0.0062	-0.0035
23	-0.64	0.55	19.39	39.33	-0.1859	0.0623	-0.0370	0.0989	-0.0175	0.0079	-0.0104	0.0093	-0.0049	0.0046	-0.0016	0.0062
24	0.29		23.77		0.0261		0.0626		0.0030		0.0044		0.0019		0.0031	
25	0.38		15.34		0.0361		0.0720		0.0019		0.0035		0.0027		0.0028	
26	0.69		13.80		0.1403		0.1132		0.0068		0.0058		0.0044		0.0042	
27	-0.66		18.28		-0.1909		-0.1145		-0.0107		-0.0075		-0.0057		-0.0049	
28	0.29		24.79		0.0193		-0.1252		0.0014		-0.0052		-0.0035		-0.0062	
29	-0.82		62.56		-0.1259		0.0230		-0.0223		-0.0152		-0.0068		0.0018	
30	0.18		7.56		0.0067		0.0432		0.0007		0.0022		0.0006		0.0012	
31	-0.67		40.31		-0.1436		-0.1077		-0.0110		-0.0094		-0.0078		-0.0068	
32	1.01		52.67		0.1164		0.0893		0.0101		0.0091		0.0076		0.0065	
33	0.97		17.68		0.1914		0.2678		0.0102		0.0134		0.0105		0.0113	
34	0.89		32.38		0.1238		-0.0206		0.0101		0.0036		0.0025		-0.0012	
35	-0.10		15.20		-0.0699		0.0790		-0.0048		0.0023		0.0013		0.0031	
36	-0.11		12.90		-0.0752		-0.0386		-0.0214		-0.0200		-0.0025		-0.0014	
37	0.18		93.08		-0.0058		0.0301		-0.0005		0.0012		0.0014		0.0029	
38	0.23		25.13		0.0127		-0.0145		0.0007		-0.0003		-0.0003		-0.0007	
39	-0.06		28.96		-0.0407		0.0357		-0.0020		0.0011		0.0012		0.0019	
40	0.36		30.95		0.0298		-0.1147		0.0026		-0.0040		-0.0025		-0.0064	
41	-0.66		24.22		-0.1686		-0.0198		-0.0162		-0.0091		-0.0047		-0.0010	
42	0.34		41.86		0.0274		0.0640		0.0033		0.0047		0.0026		0.0041	
43	0.30		17.47		0.0147		0.0506		0.0008		0.0024		0.0019		0.0021	
44	0.95		19.47		0.1770		0.1499		0.0088		0.0078		0.0069		0.0066	
45	-0.40		15.73		-0.1407		-0.0643		-0.0075		-0.0043		-0.0032		-0.0026	
46	0.25		24.94		0.0120		-0.1324		0.0010		-0.0056		-0.0035		-0.0066	
47	-1.06		78.53		-0.1397		0.0092		-0.0285		-0.0215		-0.0093		0.0008	
48	-0.02		9.96		-0.0583		-0.0217		-0.0081		-0.0067		-0.0015		-0.0007	
49	-0.78		45.57		-0.1506		-0.1147		-0.0120		-0.0103		-0.0088		-0.0077	
50	1.61		104.33		0.1410		0.1138		0.0139		0.0129		0.0133		0.0116	
51	1.20		18.94		0.2387		0.3150		0.0135		0.0167		0.0128		0.0137	
52	0.96		32.61		0.1350		-0.0095		0.0108		0.0043		0.0030		-0.0005	
53	-0.14		16.82		-0.0770		0.0719		-0.0050		0.0021		0.0013		0.0029	
54	-0.13		15.48		-0.0749		-0.0383		-0.0208		-0.0194		-0.0027		-0.0015	
Means	0.1361	0.1755	30.6992	48.8177	-0.0083	0.0122	0.0127	0.0353	-0.0025	-0.0014	-0.0015	-0.0003	0.00003	0.0008	0.0006	0.0018
Var.	0.33	1.13	558.33	735.57	0.01	0.02	0.01	0.02	0.0001	0.0003	0.0001	0.0002	0.00002	0.00009	0.00002	0.00008
t-stastic	-0.1674		-2.7851		-0.5825		-0.7051		-0.2939		-0.3546		-0.3495		-0.6067	
Level of Significance	0.868		0.008		0.564		0.486		0.771		0.726		0.729		0.549	

SI - Sharpe's index, BMS - benchmark for Sharpe's index, TM - Treynor's measure, BMT - benchmark for Treynor's measure, FM - Fama's measure

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