

Martingales and Complete Market

* *S.Vengataasalam*

** *Dr.C.Loganathan*

INTRODUCTION

The earlier assumption on the share market analysis is based on the markov property assumption. This model, to some extent, helps us to predict the risk involved in a particular industry and thereby helps us to settle down with maximum gain under prescribed limiting property.

A special type of stochastic process which is based on conditional expectations as sequence of random variables called Martingales has become a better tool to study continuous trading. This type of perspective helps one to have two types of options: one on the sampling and the other on the stopping process. The market can be regarded as a complete one in the sense that lower risk on one is compensated by higher profit on the other for investors. The study of this series has been taken up by Harrison and Pliska and we find that the martingale theory plays an important role in optional sampling and optional investment on various shares yielding low and high returns. We explain in detail the different stages of improvement on this model and their implications on the consumer's satisfaction.

A set of events form a collection of sets in the sample space which is closed under arbitrary union, finite intersection, complements and they form a σ -field. So we are left with the consideration of martingale with respect to σ -fields.

MARTINGALES WITH RESPECT TO σ -FIELDS

Until now, we have always considered conditional expectations to be expectations computed under conditional distributions. This is mostly satisfactory for expressions of the form $E[X | Y_0, \dots, Y_n]$, where X, Y_0, \dots, Y_n possess a joint continuous density or are jointly discrete random variables. However, the analysis extended to the more complex expressions like $E[X | Y_0, Y_1, \dots]$ or $E[X | Y(u), 0 \leq u \leq t]$ becomes more delicate.

The alternative and more modern approach is to define and evaluate conditional expectation, not with respect to a finite family of random variables, as we have done so far, but with respect to certain collections, called σ -fields of events. This suggests, in a natural way, a definition of a martingale with respect to a sequence of σ -fields.

The probability measure, a function P defined on F and satisfying

- (i) $0 = P[\phi] \leq P[A] < P[\Omega] = 1$, for $A \in F$ (ϕ - the empty set),
- (ii) $P[A_1 \cup A_2] = P[A_1] + P[A_2] - P[A_1 \cap A_2]$, for $A_i \in F$, $i = 1, 2$, and
- (iii) $P\left[\bigcup_{n=1}^{\infty} A_n\right] = \sum_{n=1}^{\infty} P[A_n]$,

if $A_n \in F$, are mutually disjoint ($A_i \cap A_j = \phi$, $i \neq j$).

MODIGLIANI AND MILLER

What is cost of capital to a firm in a world in which funds are used to acquire assets whose yields are uncertain; and in which capital can be gained by different media, ranging from pure debt instruments, representing money fixed claims to pure equity issues, giving holders only the right to a prodata share in the uncertain venture?

In much of his analysis, the economic theorist at least lies tended to side step the essence of this cost of capital problem by proceeding as though physical assets like bonds could be regarded as yielding sure streams. Only recently have economists began to face the problem of the cost capital cum risk seriously. Modigliani and Miller have derived the following simple rule for optimal investment policy by the firm. Regardless of the financing used; the marginal cost of capital to a firm is equal to the capitalization rate for an unlevered stream in the class to which the firm belongs.

They have considered the three major financing alternatives open to the firm-bonds, retained earnings and common stock issues and proved that in each case, an investment is worth undertaking if and only if the rate of return on investment is longer than the average cost of capital.

* *Assistant Professor*, Department of Mathematics, Kongu Engineering College, Erode, Tamil Nadu Email: sv.maths@gmail.com

** *Principal*, Maharaja Arts and Science College, Coimbatore, Tamil Nadu

The analysis developed here was essentially a comparative statistics; and not a dynamic analysis. Such analysis, as those posed by expected changes in rate of return and in average cost of capital overtime has not been treated.

BLACK AND SCHOLES

Black and Scholes made a dazzling observation; that in the idealized market, investors can duplicate the cash flow (or pay off stream) from a call option by cleverly managing a portfolio that certain only stock and bond. Since the possession of this portfolio is completely equivalent to the possession of their call option, the market value of its securities at time zero is the unique rational value for the option.

HARRISON AND PLISKA

Harrison and Pliska have taken up the study of continuous trading and developed a general stochastic model of a frictionless security market with continuous trading. Within the frame work of that model, they discussed the option pricing formula; we can use it for the study of consumption investment problems.

MODERN THEORY OF CONTINGENT CLAIM VALUATION

Let $W = \{ W_t ; 0 \leq t \leq T \}$ be a standard (zero drift and unit variance) Brownian motion on some probability space (F, \mathcal{F}, P) . Let r, μ and σ be real constants with $\sigma > 0$. Define

$$S_t^0 = S_0^0 \exp(rt), \quad 0 \leq t \leq T,$$

$$S_t^1 = S_0^1 \exp(\sigma W_t + (\mu - \frac{1}{2} \sigma^2)t), \quad 0 \leq t \leq T,$$

where the initial values S_0^0 and S_0^1 are positive constants. S^0 and S^1 are the price process for risk less security and risky security respectively. These satisfy the stochastic differential equations

$$dS_t^0 = r S_t^0 dt$$

$$dS_t^1 = \sigma S_t^1 dW_t + \mu S_t^1 dt$$

where S^1 is a geometric Brownian with rate of return $\sigma dW_t + \mu dt$, called the return process for the stock. Consider a ticket which entitles its bearer to buy one share of stock, at the terminal date T , if he wishes for a specified price of c units. Call option is equivalent to a payment $X = (S_T^1 - c)^+$ at time T .

Black and Scholes asserted that there is a unique rational value for the option, independent of one's risk attitude.

$$f(x, t) = x \Phi(g(x, t)) - ce^{-rt} \Phi(h(x, t)),$$

where

$$g(x, t) = [\ln(x/c) + (r + \frac{1}{2} \sigma^2)t] / \sigma \sqrt{t},$$

$h(x, t) = g(x, t) - \sigma \sqrt{t}$ and $\Phi(\bullet)$ is the standard normal distribution function, this unique rational value is $f(S_t^1, T)$.

The function $f(x, t)$ defined above satisfies the partial differential equation

$$\frac{\partial}{\partial t} f(x, t) = \frac{1}{2} \sigma^2 x^2 \frac{\partial^2}{\partial x^2} f(x, t) + r x \frac{\partial}{\partial x} f(x, t) - r f(x, t)$$

with initial condition $f(x, 0) = (x - c)^+$.

Define stochastic processes

$$V_t = f(S_t^1, T - t), 0 \leq t \leq T,$$

$$\phi_t^1 = \frac{\partial}{\partial x} f(S_t^1, T - t), \quad 0 \leq t \leq T,$$

$$\phi_t^0 = (V_t - \phi_t^1 S_t^1) / S_t^0, \quad 0 \leq t \leq T.$$

The market value of the portfolio held at time t is

$$\phi_t^0 S_t^0 + \phi_t^1 S_t^1 = V_t, \quad 0 \leq t \leq T.$$

The initial value of the portfolio is

$$V_0 = f(S_0^1, T)$$

and the terminal value

$$V_T = f(S_T^1, 0) = (S_T^1 - c)^+$$

is precisely equal to the terminal value of the call option. Finally applying Ito's formula we obtain

$$dV_t = \frac{\partial}{\partial x} f(S_t^1, T-t) dS_t^1 + \frac{1}{2} \frac{\partial^2}{\partial x^2} f(S_t^1, T-t) (dS_t^1)^2 + \frac{\partial}{\partial t} f(S_t^1, T-t) dt$$

Then

$$V_t - V_0 = \int_0^t \phi_u^0 dS_u^0 + \int_0^t \phi_u^1 dS_u^1 \quad 0 \leq t \leq T.$$

The right hand side represents the total earnings or capital gains which we realize on our holdings up to time t .

CONTINUOUS TRADING

We begin now with probability space (Ω, \mathcal{F}, P) and a filtration (increasing family of sub σ -algebras) $\mathcal{F} = \{\mathcal{F}_t; 0 \leq t \leq T\}$, satisfying the usual conditions \mathcal{F}_0 contains all null sets of P ; \mathcal{F} is right continuous meaning that

$$\mathcal{F}_t = \bigcap_{s>t} \mathcal{F}_s, \text{ for } 0 \leq t \leq T.$$

Let $S = \{S_t^k; 0 \leq t \leq T\}$ be a vector process whose components S^0, S^1, \dots, S^K are adapted ($S_t^k \in \mathcal{F}_t$ for $0 \leq t \leq T$), right continuous with left limits and strictly positive.

Let $S_0^0 = 1$. We could write

$S_t^0 = \exp\left(\int_0^t \gamma_s ds\right)$, $0 \leq t \leq T$, for some process γ , and then γ_t would be interpreted as the riskless interest rate at time t . Define $\alpha_t = \log(S_t^0)$, $0 \leq t \leq T$, we call α the return process for S^0 ,

$\beta_t = 1/S_t^0 = \exp(-\alpha_t)$, $0 \leq t \leq T$, calling β the intrinsic discount process for S . It will be convenient to define a discounted process $Z = (Z^1, \dots, Z^K)$ by setting

$$Z_t^k = \beta_t S_t^k \quad 0 \leq t \leq T \quad \text{and} \quad k=1, \dots, K.$$

Let \mathcal{P} be the set of probability measures Q on (W, \mathcal{F}) which are equivalent to P and such that Z is a martingale under Q , since $\beta S_0 = 1$ is a martingale under any measure equivalent to P . Elements of \mathcal{P} are called martingale measures.

We have that S^0 is a variation finite process and thus a semi martingale, that Z^k is a martingale under any $Q \in \mathcal{P}$, and that $S^k = Z^k / \beta = S^0 Z^k$, S^k is semi martingale under Q and thus also under P . Hence S is a vector semi martingale.

A trading strategy is defined as $K+1$ dimensional process $\phi = \{\phi_t; 0 \leq t \leq T\}$ whose components are locally bounded and predictable. With each such strategy ϕ we associated a value process $V(\phi)$ and a gains process $G(\phi)$ by

$$V_t(\phi) = \phi_t^0 S_t^0 = \sum_{k=0}^K \phi_t^k S_t^k, \quad 0 \leq t \leq T,$$

$$G_t(\phi) = \int_0^t \phi_u dS_u = \sum_{k=0}^K \int_0^t \phi_u^k dS_u^k, \quad 0 \leq t \leq T.$$

We interpret $V_t(\phi)$ as the market value of the portfolio ϕ_t and $G_t(\phi)$ as the net capital gains. We say that a trading strategy ϕ is self financing if

$$V_t(\phi) = V_0(\phi) + G_t(\phi), \quad 0 \leq t \leq T.$$

FORMULATION OF THE MODEL

Let us select and fix a reference measure $P^* \in \mathcal{P}$, denoting $E^*(\cdot)$ the associated expected operator. We define $\mathcal{H}(z)$ as the set of all predictable process $H = (H^1, \dots, H^K)$ such that the increasing

process, $\left(\int_0^t (H_s^k)^2 d[Z^k, Z^k]_s \right)^{\frac{1}{2}}$, $0 \leq t \leq T$, is locally integrable under P^* for each $k = 1, \dots, K$. It can be verified that $\mathcal{L}(Z)$ contains all locally bounded and predictable H and moreover $\int H dZ$ is still a local martingale for these integrands.

We now expand our definition of a trading strategy to include all predictable $\phi = (\phi^0, \phi^1, \dots, \phi^K)$ such that $(\phi^1, \phi^2, \dots, \phi^K) \in \mathcal{L}(Z)$. With $V^*(\phi) = \beta \phi S$ and $G^*(\phi) = \int \phi dZ$, a trading strategy ϕ is said to be admissible if $V^*(\phi) \geq 0$, $V^*(\phi) = V_0^*(\phi) + G^*(\phi)$ and $V^*(\phi)$ is a martingale (under P^*).

A contingent claim defined as a positive random variable X . Such a claim is said to be attainable if there exists $\phi \in \Phi^*$ such that $V_T^*(\phi) = \beta_T X$, in which case ϕ is said to generate X and $\pi = V_0^*(\phi)$ is called the price associated with X .

We give the following propositions in connection with the study of the continuous market.

COMPLETE MARKET

Proposition 1.1: (Harrison and Pliska). The unique price π associated with an attainable claim X is $\pi = E^*(\beta_T X)$.

Proposition 1.2: (Harrison and Pliska). Let X be an integrable contingent claim and let V^* be the modification of $V_t^* = E^*(\beta_T X | \mathcal{F}_t)$, $0 \leq t \leq T$. Then X is attainable if and only if V^* can be represented in the form $V^* = V_0^* + \int H dZ$ for some $H \in \mathcal{L}(Z)$, in which case $V^*(\phi) = V^*$ for any $\phi \in \Phi^*$ which generates X . Let $M(Z) \subset \Phi$ consist of all $M \in M$ can be represented in the form $M = M_0 + \int H dZ$ for some $H \in \mathcal{L}(Z)$.

Proposition 1.3: The model is complete if and only if $M = M(Z)$ Harrison and Pliska have conjectured. If P is a singleton, and then the model is complete. Their conjecture is settled by the following result. Therefore, the following statements are equivalent.

(i) The model is complete under P^*

(ii) Every martingale M can be represented in the form

$$M = M_0 + \int H dZ \text{ for some } H \in \mathcal{L}(Z).$$

(iii) P is a singleton.

By a martingale, we mean the real valued stochastic process

$$M = \{M_t : 0 \leq t \leq T\},$$

satisfying the usual definition of a martingale under the filtration \mathcal{F} and reference measure P^* .

CONCLUSION

The data of three major industries namely auto industry, medical industry and textile industry have been taken. The investor's preference on these three is calculated and in a market in which investors are engaged in these three industries finds a compactable risk free investment.

AUTO INDUSTRIES

TABLE 1: ASHOK LEYLAND

	0-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	E(x)
0-10	0.689	0.179	0.033	0.033	0.176	0.016	0.016	0.000	0.000	6.339
11-20	0.149	0.507	0.239	0.090	0.015	0.000	0.000	0.000	0.000	14.001
21-30	0.139	0.333	0.306	0.139	0.028	0.056	0.000	0.000	0.000	18.402
31-40	0.100	0.400	0.100	0.200	0.150	0.000	0.050	0.000	0.000	21.900
41-50	0.125	0.000	0.375	0.125	0.125	0.125	0.000	0.000	0.125	33.375
51-60	0.000	0.000	0.286	0.286	0.143	0.143	0.143	0.000	0.000	36.751
61-70	0.200	0.200	0.000	0.000	0.000	0.200	0.200	0.000	0.200	40.800
71-80	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
81-90	0.000	0.000	0.000	0.333	0.000	0.333	0.333	0.000	0.000	47.619
Total										219.187

Source – www.bseindia.com transactions by ASHOK LEYLAND

TABLE 2: TATA ENGG

	21-40	41-60	61-80	81-100	101-120	121-140	141-160	161-180	181-200	E(x)
21-40	0.523	0.227	0.114	0.091	0.000	0.023	0.023	0.000	0.000	40.641
41-60	0.320	0.200	0.260	0.120	0.020	0.040	0.020	0.020	0.000	53.400
61-80	0.091	0.455	0.212	0.121	0.091	0.000	0.030	0.000	0.000	56.720
81-100	0.211	0.368	0.211	0.053	0.105	0.000	0.053	0.000	0.000	54.761
101-120	0.000	0.118	0.118	0.118	0.176	0.294	0.000	0.118	0.059	104.621
121-140	0.077	0.154	0.000	0.077	0.385	0.231	0.077	0.000	0.000	91.861
141-160	0.000	0.000	0.250	0.000	0.250	0.000	0.250	0.250	0.000	116.000
161-180	0.000	0.167	0.167	0.167	0.333	0.167	0.000	0.000	0.000	84.401
181-200	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	61.000
Total										663.405

Source – www.bseindia.com transactions by TATA ENGG

TABLE.3: HERO HONDA MOTORS

	21-40	41-60	61-80	81-100	101-120	121-140	141-160	161-180	E(x)
21-40	0.500	0.286	0.143	0.071	0.000	0.000	0.000	0.000	36.700
41-60	0.278	0.278	0.167	0.111	0.111	0.056	0.000	0.000	54.401
61-80	0.000	0.313	0.125	0.313	0.188	0.000	0.063	0.000	73.682
81-100	0.111	0.056	0.389	0.278	0.000	0.111	0.000	0.056	73.321
101-120	0.000	0.100	0.000	0.400	0.300	0.100	0.000	0.400	95.000
121-140	0.000	0.000	0.600	0.000	0.000	0.000	0.000	0.400	101.000
141-160	0.000	0.000	0.500	0.000	0.000	0.500	0.000	0.000	91.000
161-180	0.000	0.200	0.000	0.000	0.400	0.000	0.200	0.200	109.000
TOTAL									634.104

Source – www.bseindia.com transactions by HERO HONDA MOTORS

MEDICAL INDUSTRIES

TABLE 4: CIPLA LTD

	00-40	41-80	81-120	121-160	161-200	201-240	241-280	281-320	321-360	361-400	E(x)
0-40	0.742	0.226	0.000	0.000	0.000	0.032	0.000	0.000	0.000	0.000	15.698
41-80	0.263	0.368	0.132	0.079	0.053	0.079	0.026	0.000	0.000	0.000	66.017
81-120	0.042	0.417	0.417	0.083	0.042	0.000	0.000	0.000	0.000	0.000	67.679
121-160	0.071	0.214	0.143	0.071	0.071	0.071	0.000	0.357	0.000	0.000	154.967
161-200	0.000	0.200	0.100	0.100	0.200	0.100	0.100	0.100	0.000	0.100	169.000
201-240	0.000	0.000	0.167	0.000	0.167	0.167	0.333	0.000	0.167	0.000	207.841
241-280	0.000	0.000	0.157	0.500	0.187	0.000	0.157	0.000	0.000	0.000	141.161
281-320	0.000	0.000	0.333	0.167	0.333	0.000	0.167	0.000	0.000	0.000	140.040
321-360	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	121.000
361-400	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	201.000
						TOTAL					1284.403

Source – www.bseindia.com transactions by CIPLA LTD

TABLE 5: RANBAXY LAB

	25-50	51-75	76-100	101-125	126-150	151-175	176-200	201-225	226-250	E(x)
25-50	0.368	0.316	0.211	0.105	0.000	0.000	0.000	0.000	0.000	51.957
51-75	0.192	0.385	0.154	0.154	0.077	0.000	0.038	0.000	0.000	68.083
76-100	0.111	0.185	0.296	0.148	0.037	0.148	0.074	0.000	0.000	89.688
101-125	0.095	0.190	0.286	0.095	0.190	0.143	0.000	0.000	0.000	88.929
126-150	0.125	0.000	0.063	0.438	0.188	0.125	0.000	0.063	0.000	107.377
151-175	0.000	0.182	0.091	0.091	0.273	0.182	0.000	0.000	0.182	128.219
176-200	0.000	0.000	0.286	0.286	0.143	0.000	0.286	0.000	0.000	118.976
201-225	0.000	0.000	0.000	0.000	0.000	0.000	0.100	0.000	0.000	176.000
226-250	0.000	0.000	0.500	0.000	0.000	0.000	0.500	0.000	0.000	126.000
TOTAL										955.229

Source – www.bseindia.com transactions by RANBAXY LAB

TABLE.6: DR. REDDY

	25-50	51-75	76-100	101-125	126-150	151-175	176-200	201-225	226-250	E(x)
25-50	0.556	0.222	0.111	0.000	0.056	0.056	0.000	0.000	0.000	49.170
51-75	0.167	0.222	0.278	0.056	0.167	0.000	0.056	0.056	0.000	84.435
76-100	0.190	0.143	0.143	0.190	0.143	0.095	0.048	0.000	0.048	93.713
101-125	0.125	0.250	0.188	0.250	0.000	0.000	0.063	0.125	0.000	91.626
126-150	0.083	0.083	0.083	0.333	0.000	0.167	0.167	0.000	0.083	119.533
151-175	0.100	0.000	0.300	0.100	0.100	0.300	0.100	0.000	0.000	110.90
176-200	0.000	0.000	0.250	0.250	0.125	0.000	0.125	0.125	0.125	135.250
201-225	0.000	0.000	0.143	0.000	0.143	0.143	0.143	0.143	0.286	168.740
226-250	0.000	0.000	0.000	0.200	0.200	0.200	0.000	0.200	0.200	160.800
					TOTAL					1014.166

Source – www.bseindia.com transactions by DR. REDDY

TEXTILE INDUSTRIES

TABLE .7: RAYMOND LIMITED

	21-40	41-60	61-80	81-100	101-120	121-140	141-160	161-180	181-200	E(x)
21-40	0.667	0.213	0.067	0.040	0.043	0.000	0.000	0.000	0.000	30.713
41-60	0.568	0.243	0.108	0.054	0.027	0.000	0.000	0.000	0.000	35.012
61-80	0.125	0.438	0.125	0.125	0.125	0.000	0.000	0.000	0.063	62.361
81-100	0.625	0.000	0.125	0.125	0.000	0.000	0.000	0.000	0.125	22.625
101-120	0.125	0.125	0.375	0.000	0.250	0.125	0.000	0.000	0.000	55.750
121-140	0.000	0.100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	41.000
141-160	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
161-180	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
181-200	0.000	0.000	0.000	0.000	0.100	0.000	0.000	0.000	0.000	101.000
					TOTAL					348.461

Source – www.bseindia.com transactions by RAYMOND LIMITED

TABLE 8: BOMBAY DYEING

	00-20	21-40	41-60	61-80	81-100	101-120	121-140	141-160	161-180	E(x)
0-20	0.814	0.186	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.906
21-40	0.179	0.308	0.385	0.128	0.000	0.000	0.000	0.000	0.000	30.061
41-60	0.105	0.158	0.368	0.105	0.158	0.000	0.053	0.026	0.026	51.874
61-80	0.105	0.263	0.263	0.211	0.000	0.105	0.000	0.053	0.000	47.255
81-100	0.000	0.071	0.214	0.286	0.143	0.286	0.000	0.000	0.000	68.180
101-120	0.000	0.100	0.200	0.100	0.200	0.100	0.100	0.100	0.100	74.900
121-140	0.000	0.250	0.000	0.000	0.250	0.250	0.000	0.000	0.250	96.000
141-160	0.000	0.000	0.000	0.200	0.400	0.200	0.000	0.000	0.200	97.000
161-180	0.000	0.000	0.000	0.000	0.250	0.000	0.250	0.500	0.000	121.000
						TOTAL				590.176

Source – www.bseindia.com transactions by BOMBAY DYEING

TABLE .9: DIGZAM LIMITED

	0-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	E(x)
0-5	0.858	0.106	0.035	0.000	0.000	0.000	0.000	0.000	0.000	1.021
6-10	0.250	0.571	0.107	0.054	0.018	0.000	0.000	0.000	0.000	5.845
11-15	0.118	0.353	0.294	0.118	0.000	0.118	0.000	0.000	0.000	10.308
16-20	0.000	0.286	0.143	0.143	0.143	0.143	0.000	0.143	0.000	17.446
21-25	0.500	0.250	0.000	0.000	0.000	0.250	0.000	0.000	0.000	8.000
26-30	0.000	0.250	0.000	0.250	0.250	0.000	0.000	0.000	0.250	21.000
31-35	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
36-40	0.000	0.000	0.000	0.000	0.100	0.000	0.000	0.000	0.000	21.000
41-45	0.000	0.000	0.000	0.000	0.000	0.000	0.100	0.000	0.000	31.000
					TOTAL					115.620

Source – www.bseindia.com transactions by DIGZAM LIMITED

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capital.WACC for paper industry, at the same time, has significantly declined with constant rate of 4.23 per cent every year to stay at 37.73 per cent in 2006-07 from 68.17 per cent in 1997-98.

- It is found that the market capitalization has been at its maximum in 1997-98 (Rs.3568.05 crores) but dropped to its minimum level in 1999-2000 (Rs.850.12 crores) after a sudden decline in 1998-99 from its level in 1997-98.
- It is found that the market capitalization ended with Rs.2706.53 crores in 2006-07, which is less than its level in the beginning year. Though insignificant, linear growth rate (LGR), which is negative, provides evidence that market capitalization for this industry has failed to show an upward trend.
- It is found that the net worth of the selected companies of the Paper industry has increased significantly at the rate of Rs.125.38 crores every year on an average to reach at Rs.2609.82 crores in 2006-07 from Rs.1415.97 crores in 1997-98.
- It is found from CV values that net worth (CV = 21.79) has been highly consistent as compared to that of market capitalization (CV = 53.49).Due to declining trend and heterogeneity in market capitalization, the value creation tends to prevail in the negative zone in 8 out of 10 years. Furthermore, negative LGR value, though insignificant, exposes the fact that market has failed to expose the real value of the Paper industry.
- It is found that MVA, on an average, is negative for low and moderate EVA groups but positive for high EVA groups. At the same time, the EVA, which stood at -54.85 for low group has increased to -22.77 for moderate group and then to 4.89 for high EVA group. This clearly reveals the existence of a positive relationship between the two. However, F value obtained from the analysis is very low and insignificant statistically, in turn indicating that difference in MVA is independent of the level of EVA for companies of the paper industry.
- It is found that there has been significant influence of EVA on MVA in 7 out of 10 years. The fit of the regression models for 1997-98 (F value = 9.08, $p < 0.05$), 1999-00 (F value = 11.99, $p < 0.01$), 2002-03 (F value = 15.77, $p < 0.01$), 2003-04 (F value = 4.73, $p < 0.10$), 2004-05 (F value = 15.90, $p < 0.01$), 2005-06 (F value = 6.41, $p < 0.05$) and 2006-04 (F value = 12.70, $p < 0.01$) is found to be significant at the required hypothetical level. This in turn provides evidence of significant association between MVA and EVA of companies under the paper sector in most of the years. But when time series data for all ten years are pooled together, the fit of the regression model becomes insignificant (F Value = 1.33, NS). From DW test values, it is understood that there exists no serial correlation in most of the years between the two variables. Therefore, overall from the above inferences, it is found that value creation based on the EVA happened on a year to year basis in respect of companies under the Paper Industry.

CONCLUSION

This study clearly revealed that there is positive relationship between EVA and MVA in the paper industry. It is concluded that the value creation based on the EVA happened on a year to year basis in respect of companies of the Paper Industry.

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