Stock Index Futures Rollover Strategies : An Empirical Study of Four Countries

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Abstract

Despite dominating transaction volume near expiration, futures rollover transactions have been little studied in developed markets or in developing markets. In this study, daily intra-market stock index futures calendar spread data for the U.S., UK, India, and China markets covering 2016 expirations formed the basis for comparing two commonly employed rollover strategies with newly devised optimal strategies based upon maximizing spread liquidity or minimizing volatility. For large positions, the optimal strategy consistently outperformed standard practitioner strategies in all four markets. For smaller initial futures positions, no performance differences between strategies were expected or found. The study also discussed practical guidelines for rolling futures positions and further research directions.

Key words : rollover strategy, calendar spreads, stock index futures, roll yield, FTSE 100, S&P 500, Nifty, A50

JEL Classification : G10, G11, G13, G14, G15

Paper Submission Date : January 20, 2018 ; Paper sent back for Revision : April 14, 2018 ; Paper Acceptance Date : April 21, 2018

As stock index futures approach expiration dates, investors seek to replace existing positions with new ones by rolling to futures having later expirations. Just prior to expirations, the trading volume of futures attributable to these rollover transactions can rise temporarily to dominate futures markets as displayed in Figure 1. To avoid the risk of two individual transactions, rollovers are almost always executed through spread orders based upon the price differences between the two futures. The efficient execution of spread orders is especially important for institutional firms and hedge funds, which often carry large futures positions in multiple markets to implement their investing and hedging strategies. Accordingly, they must be mindful of volatility and liquidity in pursing rollover strategies so that profitability can be maintained. Despite the importance of rollovers, the simple closing of one position and replacement with a similar position is little studied in either developed or developing derivative markets. Even less known is the effectiveness of rollover strategies near expiration.

This study is only the second known to focus on strategies strictly for rollovers. In it, new, optimized rollover strategies for stock index futures are defined and compared with strategies presently unstudied by scholars but commonly employed by practitioners. It is also the only known study to do so across four important index futures,

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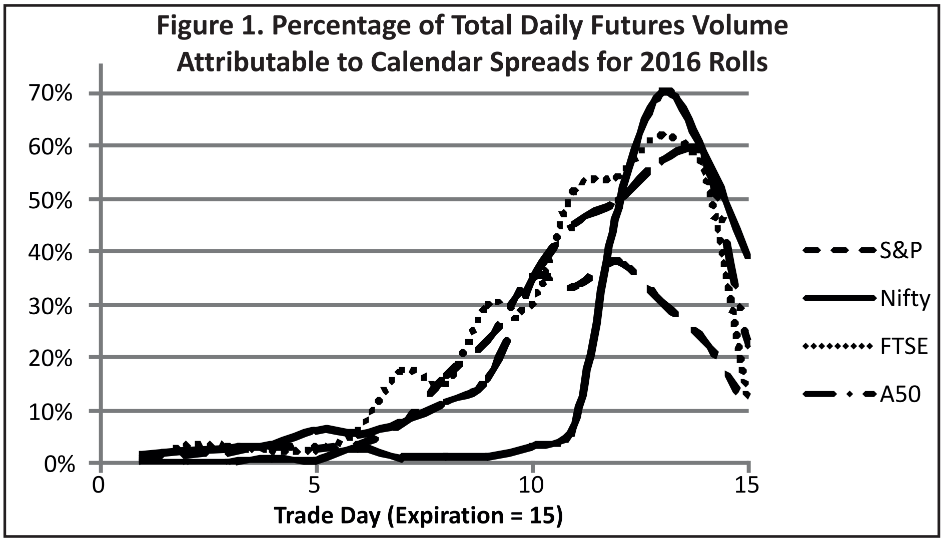


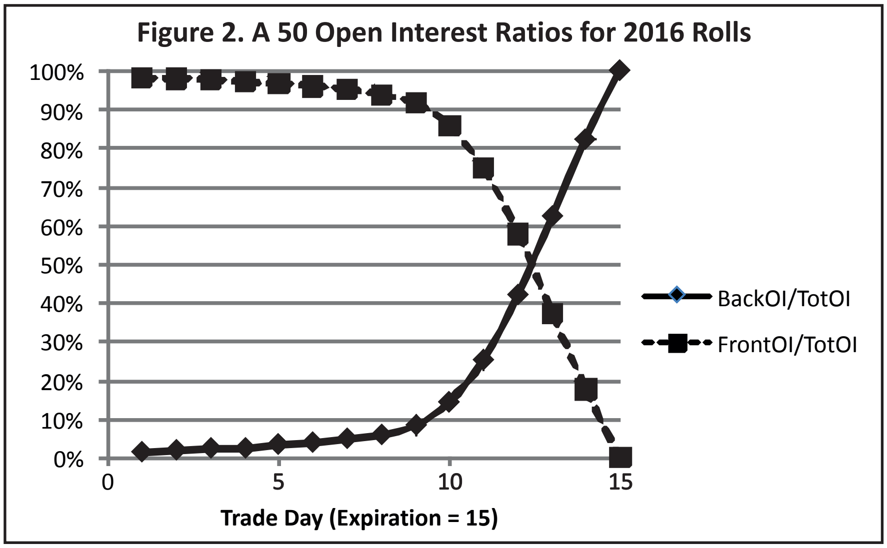
two of which are in developed derivative markets (USA, UK) and two in developing derivative markets (India, China). The results offer a passing opportunity to determine if there are meaningful differences between these two types of markets.

Literature Review

The most commonly occurring transaction in stock index futures markets is the single opening purchase or opening sale of a contract followed later by a subsequent closing trade to offset the initial position. These trades occur under a variety of market conditions and are motivated by a diverse set of investor pursuits. Most of the time, the resulting trades are without trends or sudden large changes. Accordingly, there is a little reason to study such transactions, unless they change character in a way that can lead to meaningful profit or loss. Short-lived departures from fair value, for example, can lead to profitable arbitrage at almost any time in the life of a futures contract, and an extensive literature exists on the topic of stock index arbitrage.

The second most frequently studied type of futures transaction is spreading, in which simultaneously one future is sold and one bought, with both futures either opening or both closing. Spreading is primarily the purview of the professional futures trader. The spreads so created are designed to yield a trading profit to the holder as the price difference, or spread, between two futures widens or narrows (Gatev, Goetzmann, & Rouwenhorst, 2006 ; Jones, 1981 ; Miao, 2014 ; Monroe & Cohn, 1986 ; Peterson, 1977 ; Schrock, 1971). However, these studies focused on creating profitable arbitrage positions and not on creating practical strategies for rolling futures once in place. Profitable spreading strategies naturally remain unpublished, which in part may account for why there is significantly less literature on this topic.

Spreads can be intra - market on the same commodity (Adrangi, Chatrath, Song, & Szidarovszky, 2006 ; Attaqui, Mellios, & Six, 2011 ; Abken, 1989 ; Girma & Paulson, 1999) or inter-market between different commodities (Dunis, Laws, & Evans, 2006 ; Rentzler, 1986 ; Simon, 1999 ; Wahab, Cohn, & Lashgari, 1994). Intra - market spreading in significant size requires liquidity and this liquidity is typically confined to a limited period just preceding expiration. Such spreads are normally formed between front and back month contracts and normally do not involve further deferred contracts. Concentration of liquidity near expiration severely limits the length of daily data for analysis purposes and accordingly, these spreads have attracted limited scholarly attention. Inter - market spreads, however, are among the most challenging to execute and analyze because they contain



significant basis risk, and for certain commodities can be liquid over an extended period, and they are the topic of more studies than intra - market spreads.

The least studied transaction is the intra-market rollover in which simultaneously one future is closed and another is opened. These rollovers are accomplished with calendar spread orders but are not to be confused with spread orders in which both legs are for opening or both for closing transactions. Rollover transactions arise naturally from already established positions that to be maintained must be reestablished, or rolled over, as the near-term future approaches expiration (Mouakhar & Roberge, 2010 ; Taylor, 2015) as for example, in the creation of synthetic index funds (Slivka & Li, 2010). Normally in the period immediately preceding an expiration, open interest in the front month stock index futures contract is rolled to the back month contract almost entirely by calendar spread orders. This behavior can be inferred from the profile of front, back, and total open interest displayed in Figure 2 for 2016 rolls of the A 50 futures. Total open interest (front plus back months) remains relatively constant, while the composition changes smoothly as spreads orders are completed.

As important a transaction type as rollovers are, their study is made difficult for three primary reasons. First, as observed, intra-market rollovers typically occur only in the final trading days prior to expiration and then only with a quarterly or at most, monthly frequency. This characteristic significantly limits the amount of data available for statistical analysis for any given expiration and challenges quantitative measures whose accuracy relies upon having a lengthy time series. Next, rollovers are structurally and practically so simple that they afford little opportunity for quantitative modeling which, for many scholars, is often a necessary feature to be attractive for study. Third, in some important markets, a separate spread market does not exist or is not permitted by regulators. One notable example is in the market for China's CSI 300 stock index futures. Despite these limitations, rollover transactions are worthy of special attention as transaction volumes can dominate market volumes (Figure 1). Only one other study of rollover strategies has been completed (Slivka, Yang, & Wan, 2017) and that was limited to the India market. Within the UK, U.S., China, or other markets, no known studies of rolling strategies have been identified.

This paper deals only with intra - market rollover transactions in four countries : the U.S., UK, India, and China. It seeks to answer four questions important to institutional futures practitioners with a significant daily futures presence in these markets :

(1) Comparing mature futures markets in developed countries (USA, UK) with developing futures markets in emerging economies (India, China), are there important timing differences in rollover transaction volumes and open interest as expiration dates approach ?

(2) Are there consistent relationships between calendar spread price, volume, and volatility near expiration dates, knowledge of which might improve rollover performance ?

(3) Given a typical profile of calendar spread volume vs. trading day, is there a single most favorable day or set of days on which to execute rollover orders ? Such a strategy is commonly sought by practitioners.

(4) Compared to the two commonly employed rollover strategies of executing all rolls on a single day (stack roll) or rolling an equal number of contracts per day over a chosen period (even roll), are there alternative, potentially superior trading strategies for rolling futures positions ?

Data and Methodology

(1) Data Description : To answer the above mentioned four questions, Bloomberg daily transaction data was gathered for four stock index futures contracts covering all 2016 expirations (Table 1). For the China market, the FTSE A 50 futures were chosen for analysis since a spread market with spread data exists for this contract, but not for China's primary CSI 300 futures. Expirations are monthly for the Nifty and A 50 futures and quarterly for the FTSE 100 and S&P 500 E-mini contracts.

Table 1. Four Countries' Daily Data Captured (2016)

Stock Index Levels
Country LIBOR (1 - 180 days)
Dividends in index points
Single Futures Prices, Volumes, Open Interest
Calendar Spread Prices, Volumes

Note: Indices are Nifty, A 50, FTSE 100, and S&P 500

To address the first practitioner question, historical profiles were constructed for calendar spread volumes to determine if a uniform period existed for all four countries over which comparisons could be made. Volume and open interest profiles also provided a guide to the effective trading days for conduct of a rollover strategy. A correlation analysis using spread price, volume, and volatility facilitates the search for helpful, potentially systematic relationships. To address question (3) requires the search for a suitable measure for defining "most favorable". One candidate often cited in spread studies was the roll yield. However, Parkinson volatility (Parkinson, 1980) for spreads provided a better alternative. For question (4), numerical results for spread characteristics were used to construct optimized trading strategies for comparison with results from rolling all contracts on a single day or rolling an equal number per day.

(2) Terminology : Terminology used in studies of futures spreading and rollovers is typically not uniform. For clarity, the following terms are utilized to provide clarity and facilitate comparisons across the four chosen countries.

(i) Calendar Spread, Front, Back, and Deferred Month Contracts : Rollover of a futures position using a calendar spread in this study refers to the closing of a contract having the shortest possible time to expiration (the front month) with the simultaneous opening of a contract having the next nearest time to expiration (the back month). Contracts with later times to expiration are deferred contracts and spread volumes with these futures were found to be tiny or absent. These spreads were not considered in this study. Total open interest is the sum of front and back month open interest, since deferred contract open interest was normally nil. Technically, calendar spreads exist combining front and all deferred month contracts, but this study limits the definition to front and back months.

(ii) Roll Analysis Period (RAP) : The roll analysis period (RAP) is defined in this study to be 15 sequential trading days prior to and including expiration day of the front month contract. This number of days was found to comfortably include all meaningful calendar spread trading volume.

(iii) Roll Plan : A roll plan is simply the number of contracts chosen in advance for rollover each remaining day of the RAP.

(iv) Futures Maturity Curve : Is the curve connecting front, back, and deferred month contract prices vs. days to expiration.

(v) Implied Financing Rate or Implied Forward Rate : The forward rate, rf^*, implied in a spread market price is termed as the implied financing rate or implied forward rate. This rate can differ from the forward rate rf, computed directly from the yield curve (Slivka, Li, & Zhang, 2011).

(vi) Cheap and Rich Rolls / Negative and Positive Carry : It is easily shown that when rf^* is less than (more than) rf, the calendar spread price quote is less than (more than) its theoretical fair value and the roll is said to be "cheap" ("rich").

(vii) Pace of Roll Ratio : The Chicago Mercantile Exchange (CME) defines the pace of roll ratio as the ratio of the back month open interest to the total open interest across all listed contracts. The pace of roll curve rises steadily as expiration date approaches as in Figure 3.

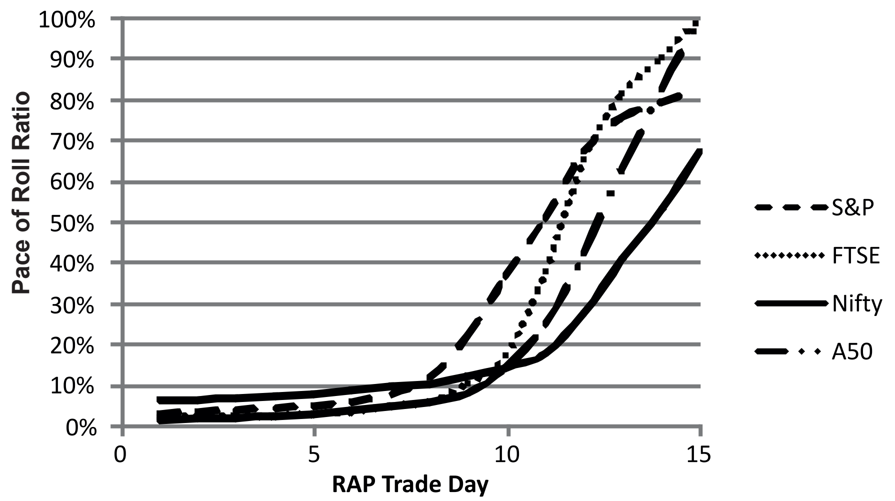
(viii) Roll Yield (RY) : Roll yield is a term used by practitioners and authors to refer to the hypothetical results of executing a calendar spread.

(ix) Zero Arbitrage Band : The price band surrounding futures fair value within which index arbitrage is not profitable is called the zero arbitrage band (ZAB). Within this band, the future can be operationally considered to be efficiently priced.

Empirical Analysis, Results, and Discussion

(1) Analysis of Open Interest and Volume : The Figure 1 shows the percentage of total daily futures trading volume (front and back months) during the RAP attributable to calendar spreads. Each country profile was constructed by averaging over the complete set of 2016 rolls. Each calendar spread accounted for two contracts traded. Between RAP days 8 and 11, the percentage rose above 10% and normally peaked between RAP day 12 and 14. These results could not have been anticipated since exchanges do not publish such analysis. It appears that a RAP of 15 days is adequate for capturing and analyzing spread data, whether the country in question is developed or developing. The peaks in these spread profiles are between 40% and 70%, emphasizing how

Figure 3. Pace of Roll vs. RAP Trade Day for 2016 Rolls in Four Countries



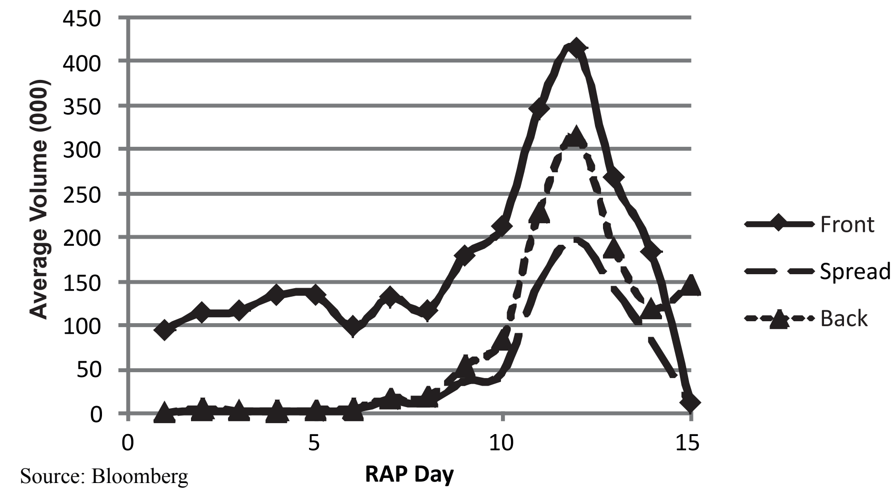
important spread transactions are as expiration dates approach.

The profiles in Figure 1 can be inferred from Figure 2, which contains open interest data normally available from exchanges and data vendors. As a typical example, the transition of open interest for 2016 A 50 futures is displayed. Results for the other three countries are similar. The total open interest as a percent of its value for the first RAP day is largely stable, suggesting that closing front month contracts are offset almost one for one by opening back month contracts. Such behavior is typical for investors maintaining a longer-term position by simply exchanging front for back months. As this process unfolds, it is assumed that investors utilize the existing exchange spread markets rather than taking the unnecessary risk of executing independent front and back month in separate transactions. As back month volume rises, it naturally facilitates the rise of spread volumes.

Figure 3 displays the average pace of roll for 2016 expirations across four countries. While the pace of roll can be found on websites for individual U.S. and UK contracts, the National Stock Exchange of India and the Singapore Exchange provide only the raw data from which the ratio can be computed. The historical pace of roll curves once again verify that there is a consistent period (RAP) prior to front month expiration during which most rollovers occur. In all four countries, RAP of 15 trading days is quite adequate for study of changes in open interests with most activity occurring during the last 5 to 7 trading days. Investor rollover behavior appears generally consistent, whether the country is considered developed or developing. Volume profiles for all countries are also generically similar. An example is displayed in Figure 4 for the 2016 FTSE rolls. Figures 1 through 4 collectively confirm that a RAP period of 15 trading days is also adequate for analysis of calendar spread volumes with most changes again occurring within the last 5 to 7 days.

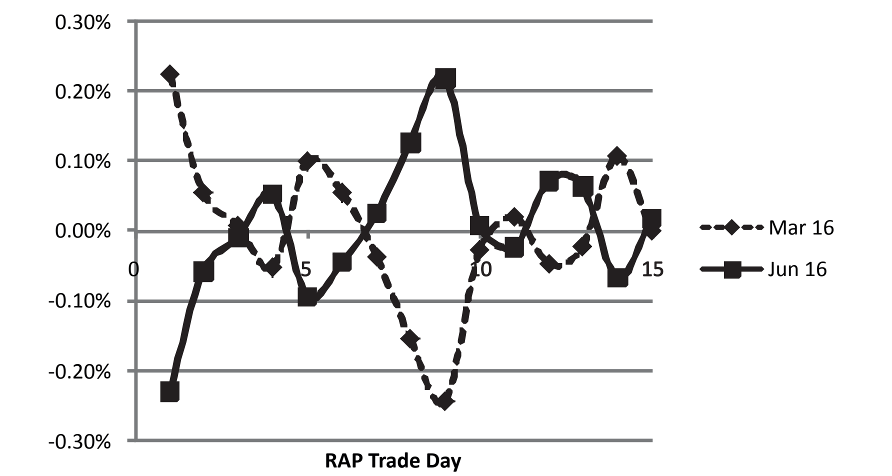
By design, stock index futures converge to the underlying index level at expiration, so price deviations from theoretical fair value are minimal as expiration approaches. During the RAP, any index futures difference from fair value that lies inside the future's zero arbitrage band will make index arbitrage unprofitable. Deviations outside the ZAB will invoke arbitrage that pushes prices back inside the ZAB. The width of the ZAB in developed markets is typically delineated by thresholds above and below a fair value of approximately 0.5%. In this study, with rare exception, the front and back month day - end futures prices for all countries are consistently inside a ZAB of 0.5%, leading us to conclude that the calendar spreads are efficiently priced during the RAPs. The Figure 5 provides an example of futures percentage deviation from fair value during the FTSE 100 March 2016 roll. Being so close to expiration, this outcome is not surprising since the convergence property of futures assures that the

Figure 4. Front, Back, and Calendar Spread Volumes vs Trade Day for 2016 FTSE Rolls



Source: Bloomberg

Figure 5. Percent Futures Deviation from Fair Value for March 2016 FTSE 100 Roll



front month price remains very close to fair value with so few days left to expiration.

(2) Parkinson Volatility : While the ordinary method for calculating volatility is to compute the standard deviation from closing prices, it is not statistically the strongest method. When sample sizes are small, it is better to adopt an alternative measure. With data limited to a few RAP days, the Parkinson volatility measure proves to be a better choice for computing calendar spread price dispersion. This method uses daily high and low prices instead of closing prices only.

$$Parkinson\ Volatility = Vp = \sqrt{\frac{1}{N}}\sqrt{\frac{1}{4Ln(2)}\sum_{i=1}^{N}\left[Ln\left(\frac{h_i}{l_i}\right)\right]^2} \qquad (1)$$

h_i = High price on RAP Trade Day I,
l_i = Low price on RAP Trade Day I,
N = number of calculation days.

Another alternative choice for a volatility measure, and one that is statistically even more powerful than Parkinson's was developed by Yang and Zhang (Yang & Zhang, 2000). This measure has a low error of estimation compared with close-to-close and Parkinson volatility. However, the required data for this computation on calendar spreads is not normally available on Bloomberg and other data sources.

The Figure 6 shows the daily Parkinson volatility *(Vp)* for 2016 E-mini and FTSE rolls as expiration date approaches. A rising volatility in the U.S., India, and China markets is consistently observed and corresponds fittingly to trader empirical observations that volatility rises in the final days of contract life. A linear fit to a rising volatility profile is found to be statistically significant at the 95% confidence level for all but the FTSE contract which has a slope consistent with zero.

The Figure 7 shows the 2016 calendar spread volatility for the Nifty and A 50 futures during the final six RAP trade days. Both India and China volatilities were regularly above those for the U.S. and UK. These profiles are appropriately consistent with the general knowledge that developing markets have higher volatilities than developed markets.

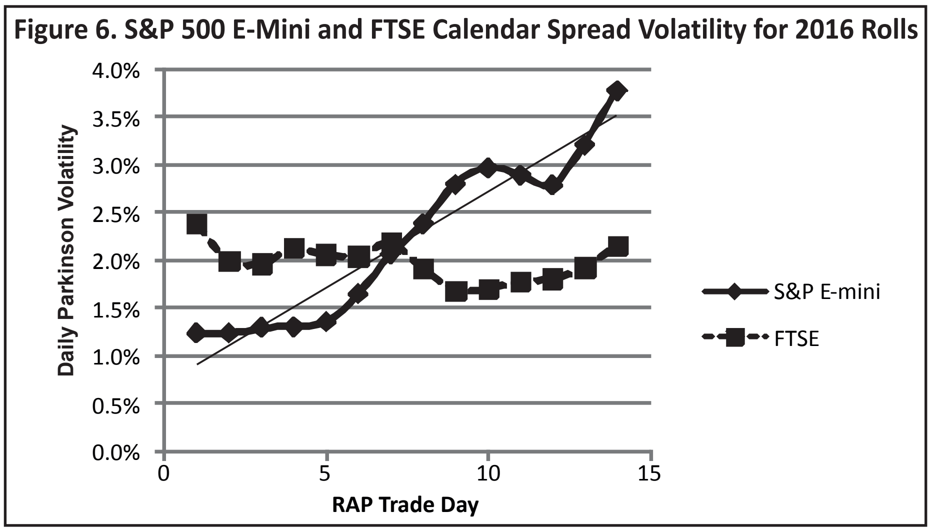


Figure 6. S&P 500 E-Mini and FTSE Calendar Spread Volatility for 2016 Rolls

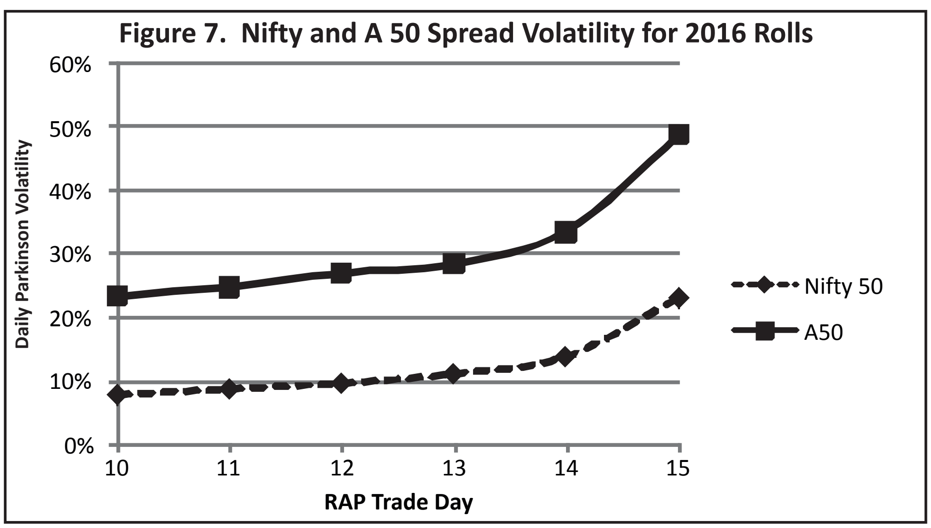


Figure 7. Nifty and A 50 Spread Volatility for 2016 Rolls

(3) Correlation Analysis : To discover possible relationships between important variables, correlations are computed for each roll in each country using daily values of the following elements :

- RAP trade day
- Parkinson volatility *(Vp)*
- back volume
- spread volume
- front volume
- spread settle price
- roll yield

Since the maximum number of data points for any individual roll was limited to the RAP (15 trade days) and only a fraction of the RAP held active days for spread transactions (see Figure 1), correlation values are treated with caution. Given the generally small sample size, correlation results are considered suggestive of relationships rather than confirmed as being statistically definitive. Absolute correlations at or above 0.9 are considered strong, between 0.70 and 0.9 as moderate, and below 0.7 as weak. Spread settle price and roll yield are clearly related to one another by definition as each includes the price difference between front and back month contracts. However, no meaningful correlation could be found between these two variables and any of the others. Accordingly, the analysis proceeds with the remaining five variables.

For example, using the weak, moderate, and strong classification of correlations, the correlation matrix for FTSE variables was computed and weak values eliminated, leaving the balance to be examined for potentially useful relationships. The matrix in Table 2 covers all 2016 FTSE rolls and suggests affiliations among the test variables. Except for the relationship between RAP trade day and Vp (discussed later), the FTSE matrix resembles the matrices for the other three countries in this study.

Table 2. Correlation Matrix 2016 FTSE Rolls

	Trade Day	*Vp*	Back Volume	Spread Volume	Front Volume
Trade Day	1		0.76		
Vp		1			
Back Volume	0.76		1	0.94	0.79
Spread Volume			0.94	1	0.93
Front Volume			0.79	0.93	1

Three of the correlations in this matrix are easily understood with reference to Figure 4 where front, back, and spread volumes rise and fall together over the RAP. Back to front volume correlation is seen to be moderate. However, the relationship is far stronger when the RAP is divided into rising and falling volume segments (Figure 4). For example, during the first 12 RAP trade days, the back to front volume correlation actually rises to 0.99 and is degraded only when declining spread volume is included in the balance of days before expiry. Back to spread and front to spread volume correlations are seen to be strong. This is understandable given that spread liquidity depends mutually on the availability of front and back month volumes. For spread volume to rise, back volume must first rise to complement available front month volumes and facilitate spread trading. Back month trading volume begins to build on RAP trade day 9 from a small base in Figure 4, increasing rapidly thereafter. This liquidity growth in turn drives the volume of spreads as reflected in the correlation matrix.

These correlations confirm common relationships known among futures practitioners executing rolls at expirations and complete the explanations for correlations as observed in Table 2. In summary, the correlation matrix across FTSE rolls taken together with the volume and open interest profiles (Figures 2 - 6) reveal that :

- FTSE volatility is flat vs RAP trade day, but rises for other futures (Figures 6, 7).
- Rising back month volume facilitates rising spread volume and liquidity (Figure 4).
- Back, front, and spread prices all remain very close to their fair values near expiration (Figure 5).

Correlation results are consistent conclusions across index futures in this study.

The results for roll yield, in particular, deserve some comments. Roll yield is a term used by both practitioners and scholars to refer to the hypothetical return results of executing a calendar spread. The definition of this term is not consistent across sources, and is therefore, source dependent. For example, Mouakhar and Roberge (2010) defined roll yield as spot index price minus the price of the back month. Using prices for front and back months as $F1$ and $F2$, Spurgin (1999) defined roll yield as $F2 - F1$ times the fraction of contracts rolled on a single chosen day. Anson (1998) defined roll yield as the difference in a single futures price at time $t+1$ and at a prior time t. S&P defined roll yield as a spread percentage of the back month futures price (S & P Dow Jones Indices, 2016).

$$RY = (F1 - F2)(12/d) / F2 \qquad (2)$$

where, d is the number of months between the front and back month and is three for futures with a quarterly expiration cycle. This is the most widely used definition and is used primarily in the commodity futures markets. Campbell & Company (2014) defined the roll yield as the cumulative value of excess benefits from trade date to expiration date of the back month. These differing definitions each represent an attempt to quantify perceived potential benefits from rollover transactions. None represent a true realized profit or loss generated on the day of the roll, making the term "yield" a misnomer and of highly questionable value.

To understand the concept of roll yield, true profit and loss calculations must be suspended in favor of a hypothetical transaction in which the futures maturity curve remains static from trade date looking ahead. Next, assume that there is an actual revenue pickup on trade date of $F1 - F2$. Since the back month replaces a holding in the front month, the hypothetical investment becomes $F2$. Assume further the holding period until execution of the next roll is the same as that between the front and back month expiration days. For quarterly expiring futures, such as on the S&P and FTSE, this is equivalent to three months. Then a hypothetical annualized "yield" can be calculated as in equation (2).

It should be clear with these multiple assumptions that RY is not a true yield of any sort. Neither are any of the roll yields defined by other authors. Many authors are confused on this practical point and treat the roll yield as a realizable value, which it is not. Even if a positive roll yield were realizable, no attention is given by authors to whether a rollover results in a pickup of revenue over a calendar spread fair value, a necessary condition for positive profits. Any such excess value - added from a rollover occurs from a long position only if the rolls are cheap ($rf^* < rf$).

(4) Rollover Trading Strategies : Two rollover strategies commonly used by investors seeking to roll front month positions are the even roll and stack roll strategies. These two rollover strategies are described before developing an alternative optimal rollover strategy that uses the information content of volume profiles and correlations.

(i) Even Roll Strategy : In this strategy, an investor chooses a start day typically within the RAP, and divides the total front month open position by the number of remaining RAP days. Assuming daily liquidity allows, an equal number of contracts are then rolled each day. To avoid adversely influencing the spread's bid-ask, the start date should be chosen so that the intended daily trade size is small relative to total spread volume. Typically, this limits the spread volume to be disposed of to 5% - 20% of the daily spread volume.

(ii) Stack Roll Strategy : A single day in the RAP is chosen, and all front month contracts held on that day are rolled to the back month. For example, once the back month volume begins to rise rapidly, there is likely to be satisfactory liquidity to consider rolling an entire front month position (stack). To avoid incurring unwanted costs by adversely moving spread price or by widening the bid/ask spread, the planned daily transaction volume is chosen by traders to be limited to a specific percentage (often 5% - 20%) of the historical daily traded spread volume depending upon market conditions.

The historical calendar spread volume profile in Figure 4 suggests a trading strategy for maximizing the aggregate weighted average liquidity experienced while rolling contracts during a RAP. The maximization should be subject to the practical constraint of keeping trading volume below w% of its historical value, where w is normally chosen based upon market conditions and size of contracts to roll. A range of 5% to 20% is typical.

The historical rising calendar spread volatility in Figures 6 and 7 suggests yet another trading strategy, this one for minimizing the aggregate accumulated volatility experienced over the RAP. Once again, the practical constraint of choosing w between 5% and 20% should be observed to avoid bid-ask distortions in spread pricing.

(iii) Maximum Liquidity Roll Strategy L : In their pure form, even and stack roll strategies make no assumptions about whether there is information content in the historical pace of roll or calendar spread volume profiles. However, these profiles suggest varying liquidity during the RAP which may be of value. It is normally the case in markets that investors generally prefer higher to lower liquidity. As a proxy for liquidity, the daily traded volume of spreads can be used by averaging 2016 roll behavior.

Let,

N = number of front month contracts to roll during the RAP,
NT = total of all spread contracts typically traded during the RAP,
Nj = number of contracts to roll on day j,
j = Trade day (j = 1- 15) during the RAP,
Φj = Historical percentage of calendar spreads traded on trade day j,
Vj = Historical spread volume on trade day $j = \Phi j \times NT$,
$Pj = w \times \Phi j$ = Adjusted percentage of calendar spreads traded on trade day j,
w less than 20% chosen by the investor depending upon market conditions,
$NMAXj$ = maximum spread volume allowed to trade on trade day $j = w \times \Phi j \times NT$,
$Aj = Nj \times Vj$ = Aggregate liquidity encountered on trade day j,
Average Φj values appear in Table 3 and Figure 8 for all four contracts.

Strategy L then becomes the set of Nj that satisfy the following objective function and constraints :

$Maximize\ AT = \Sigma_1^{15} Aj$

Subject to

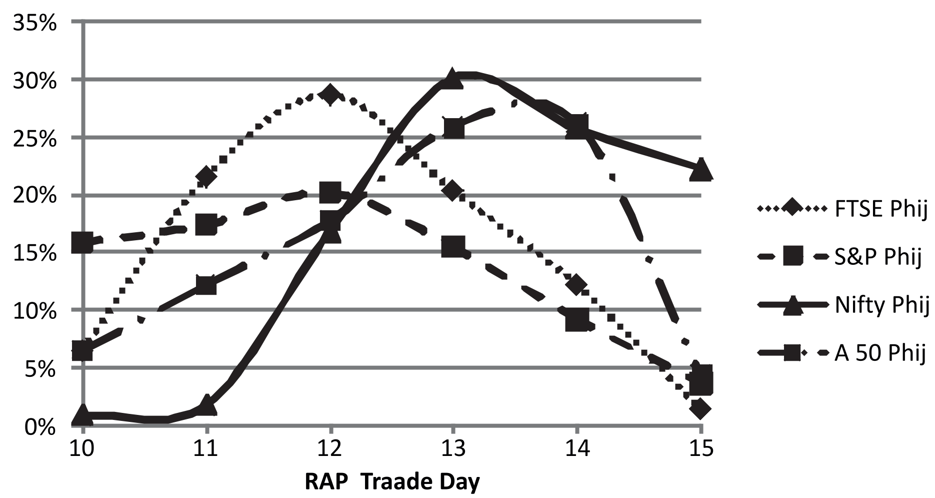
$$Nj <= w \times \Phi j \times NT = NMAXj \quad (3)$$

$$\Sigma_1^{15} Nj = N$$

Table 3. Average Historic Percentage of Traded Spread Volume

Trade Day	S&P Phi_j	FTSE Phi_j	Nifty Phi_j	A 50 Phi_j
1	0.2%	0.0%	0.0%	0.2%
2	0.4%	0.3%	0.1%	0.3%
3	0.5%	0.3%	0.1%	0.4%
4	0.8%	0.2%	0.2%	0.4%
5	0.8%	0.3%	0.2%	0.7%
6	1.0%	0.5%	0.9%	0.6%
7	2.0%	1.9%	0.3%	1.0%
8	4.8%	1.4%	0.4%	1.4%
9	8.1%	5.0%	0.4%	2.3%
10	15.8%	6.4%	0.8%	6.5%
11	17.4%	21.4%	1.8%	12.1%
12	20.1%	28.6%	16.8%	17.8%
13	15.5%	20.4%	30.1%	25.8%
14	9.1%	12.1%	25.8%	26.1%
15	3.6%	1.3%	22.2%	4.4%

Figure 8. Percent of Spread Volume Traded on RAP Days



The resulting values of *Nj* define a roll plan containing the number of contracts to roll on trade day *j*. *AT* is the value of the total aggregate weighted average liquidity for Strategy L for a choice of *w*. The behavior of Strategy L is best illustrated by application to FTSE futures. For combinations of *w* and *N*, where *N* is greater than the largest *NMAXj*, the *Nj* will be forced to spread out concentrating first near the highest value of Φ*j* (FTSE Trade Day 12) and then populating the balance of trade days consistent with the constraints. For combinations of *w* and *N*, where *N* is less than all *NMAXj*, the *Nj* will be zero except for the trade day *j* on which Φ*j* is a maximum. All other combinations of *w* and *N* will have solutions in between these two cases.

For an institutional investor position with *N* = 1000 and chosen *w*, the Table 4 contains Strategy L roll plan solutions for permitted *Nj* and aggregate liquidity for the FTSE September - December 2016 roll. The unlikely choice of *w* = 0.15% is simply to illustrate the behavior of Strategy L solutions. In practice, this strategy increases in value as the market in which it is applied becomes less liquid, forcing smaller values of *w*. Such is sometimes the case in lesser developed countries than covered in this study.

Table 4. September - December 2016 Roll Strategy Solutions for *Nj* and *AT*

RAP Day	Strategy L			Even Roll			Stack Roll*		
j	*w* = 0.15%	1%	5%	*w* = 0.15%	1%	5%	*w* = 0.15%	1%	5%
1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0
9	66	0	0	0	0	0	0	0	*
10	12	0	0	0	0	0	0	0	*
11	296	0	0	200	200	200	0	*	*
12	325	1000	1000	200	200	200	0	*	*
13	189	0	0	0	200	200	0	*	*
14	113	0	0	0	200	200	0	0	*
15	0	0	0	0	0	200	0	0	0
AT	**221.52**	**291.91**	**291.91**	**Infeasible**	**Infeasible**	**168.08**	**Infeasible**	**242.57**	**155.80**

Note : 5* RAP Days on which all 1000 contracts might be rolled.

Regardless of the choice of *w*, Strategy L results are superior, being 20% to 80% better than alternative even and stack roll strategies. For values of *w* below 1%, even and stack rolls often become infeasible since the sum of permissible *Nj* fail to be equal to *N*. Stack roll and even roll strategies then may not be successfully implemented at very low values of *w*. For $w > 1\%$, stack rolls allow multiple days on which there is ample liquidity for execution of all contracts at once. To fairly compare stack roll performance, the *AT* value is taken as the average across all permissible RAP trade days.

(iv) Minimum Volatility Roll Strategy V : In markets where spread volatility is observed to rise as expiration approaches (Figures 6, 7), an optimized volatility strategy can be devised. Among the four markets in this study, such a strategy would best apply to the S&P E-mini, Nifty, and A 50 futures. In this form, Strategy V minimizes weighted average volatility aggregated over the RAP, subject to liquidity (historical trading volume) constraints.

Let,

N = number of front month contracts to roll during the RAP,

NT = total of all spread contracts typically traded during the RAP,

Nj = number of contracts designated to roll on day *j*,

j = trade day (j = 1-15) during the RAP,

Φj = Historical percentage of calendar spreads traded on trade day *j*,

$Pj = w \times \Phi j$ = Adjusted percentage of calendar spreads traded on trade day *j*,

w less than 20% chosen by the investor depending upon market conditions,

Vpj = Historical Parkinson volatility on trade day *j*,

$NMAXj$ = maximum spread volume allowed to trade on trade day $j = w \times \Phi j \times NT$,

$Aj = Nj \times Vpj$ = Aggregate volatility measure experienced on trade day j.

Strategy V then becomes the set of Nj that satisfy the following objective function and constraints.

$$\text{Maximize } \overline{VP} = \Sigma_1^{15} \left(\frac{Nj}{N}\right) Vpj$$

Subject to

$$Nj <= w\, \Phi j\, NT = NMAXj$$

$$\Sigma_1^{15} Nj = N \qquad (4)$$

The roll plan for trade day j is simply the set of Nj. For a given choice of w, $\overline{VP}$ is the weighted average volatility measure expected in Strategy V. For combinations of w and Φj, where N is less than all $NMAXj$, N will simply concentrate at the lowest volatility trade day (trade day 7 for Nifty and 1 for A 50). If N is greater than the largest $NMAXj$, Nj will concentrate first on the smallest value of Φj and then populate the balance of trade days in increasing order of Φj consistent with constraints.

As an example, consider an institutional investor having an A50 position of N = 1000 seeking to roll from March 2016 to April 2016 with a variable value of w. The Tables 5 and 6 display Strategy V solutions for March 2016 A50 roll plans and average volatility for the March - April 2016 roll, assuming the historical March volatility profile. As w rises, the Optimal Roll Strategy V increasingly allows concentration of trades on RAP days 1, 2, 3, which have the smallest volatility.

On the other hand, by requiring daily trading at a constant level, the equal roll strategy must use many later RAP days having higher volatility to satisfy the constraint that the sum of all daily trades equals the initial position of

Table 5. A 50 March 2016 Roll Strategy Solutions for 1000 Contracts

	Optimal Roll			Even Roll			Stack Roll*		
Trade Day *i*	***w* = 5%**	**10%**	**20%**	**5%**	**10%**	**20%**	**5%**	**10%**	**20%**
1	93	186	373	0	0	0	0	0	0
2	53	105	211	0	0	0	0	0	0
3	148	297	416	77	77	77	0	0	0
4	200	400	0	77	77	77	0	0	0
5	365	12	0	77	77	77	0	0	*
6	0	0	0	77	77	77	0	0	*
7	141	0	0	77	77	77	0	*	*
8	0	0	0	77	77	77	0	*	*
9	0	0	0	77	77	77	0	*	*
10	0	0	0	77	77	77	*	*	*
11	0	0	0	77	77	77	*	*	*
12	0	0	0	77	77	77	*	*	*
13	0	0	0	77	77	77	*	*	*
14	0	0	0	77	77	77	*	*	*
15	0	0	0	77	77	77	*	*	*
	1000	1000	1000	1000	1000	1000			

Note : * RAP Days on which all 1000 contracts might be rolled.

Table 6. Aggregated A 50 Volatility for March - April 2016 Roll

w	Strategy V	Even Roll	Stack Roll
5%	9.02%	16.32%	23.81%
10%	8.39%	16.32%	19.45%
20%	7.90%	16.32%	17.71%

1000 contracts. All permissible days on which the full number of contracts can be traded using a stack roll are indicated in Table 5 with an asterisk. The average volatility measure for this strategy in Table 6 is taken to be the average of over permitted days. Even when choosing the first permissible stack roll day Strategy V, the optimal roll choice outperforms the stack roll.

The average roll strategy performance for all 2016 A 50 RAPs similarly show that the optimized roll Strategy V provides superior performance over even and stack roll strategies for the entire chosen range of w. The results displayed here are consistent with those for the U.S., India, and China markets, lending confidence that optimized Strategies L and V bring value-added to the search for improved roll strategies.

Research Implications

The stack roll strategy is sometimes selected by traders for implementation on exactly the same RAP trade day for every expiration because it is not certain that the chosen RAP day will coincide with the minimum volatility. The effectiveness of a stack roll strategy is in question, especially in a nearly flat volatility market such as occurred for the FTSE futures. For markets where volatility rises with RAP Trade Day (U.S., India, China), there is a benefit to transacting earlier in the RAP than later. However, stack rolls earlier in the RAP may be limited by lower liquidity, which forces trades later into the RAP. Depending upon market conditions, the choice of *w* may also move permissible trades later into the RAP. Regardless of the choice for *w*, the stack roll underperforms the optimized strategies.

To be consistent in strategy comparisons, when an even roll strategy has a daily volume to trade that violates the constraint imposed by a choice of *w*, no trade is allowed and the remaining trades per RAP day are recalculated for potential application of the strategy on the following day. Such situations typically occur early in the RAP, making strategy implementation infeasible and shifting permissible rolls to later RAP dates having higher volatility. Due to this feature and that an even roll strategy ignores the information content in the volume and volatility profiles, it underperforms the optimized strategies.

The roll plans created using the optimized Strategies L and V directly use the information in spread volume and volatility profiles, and their superior performance suggests that they are worthy of further development and application. It is not clear why FTSE futures spread volatility during the RAP remained flat ; whereas, volatility rose in the other three markets in this study. It may be that the FTSE futures expiry settlement process effectively moderates disruptive index pricing by using a brief intra - day auction, known as the exchange delivery settlement process (EDSP). This process matches supply and demand for each of the 100 shares in the index during a trading session at 10 am London time and lasting about 5 minutes. The resulting share prices are used to compute the final index settlement price. This matching process effectively dampens pricing volatility and may be the source of the difference in spread volatility behavior near expiration dates. If so, then regulators and exchanges in other countries with stock index futures may wish to consider adoption of similar rules to mitigate last trading day volatility. This would be especially true for the Singapore Exchange on which the A 50 futures trade.

Summary and Conclusion

The four stated objectives of this study have been met when dealing with the stock index futures markets in the U.S., UK, India, and China. In all four markets, a roll analysis period (RAP) of 15 trading days prior to expiration spans virtually all the changes in calendar spread volume and open interest, making it an attractive period during which to conduct rolling of front month futures contracts.

Typically, in all four markets, 90% of spread trading volume is concentrated within 5 days prior to and including expiration. The volume rises rapidly, reaches a peak, and about 3 days prior to expiration, enters a decline. Whether a market is developed (U.S., UK) or developing (India, China) appears to make no meaningful difference in spread trading and open interest profiles. This suggests it may be that investor trading behavior is standard across markets independent of their developed or developing status.

Spread price and roll yield evidence no significant correlation with other study variables. Front, back, and spread volumes are correlated as expected. A relationship between RAP trade day and Parkinson volatility is found to exist and is statistically significant for U.S., India, and China markets, but not for the UK. The UK's Exchange Delivery Settlement Process may explain this difference. No other consistent relationships are evident from correlations between other variables in this study.

In the U.S., India, and China, spread volatility is confirmed to rise as expiration approaches. Using this behavior and the historical spread volume profile, a strategy minimizing accrued volatility subject to a practical volume constraint (Strategy V) is constructed. If minimizing spread volatility during the RAP is the investor's objective, then Strategy V is highly desirable. Seeking to roll a fixed percentage of contracts on each RAP day in these three markets (even roll strategy) or to roll all contracts on a single day (stack roll strategy), each underperformed Strategy V by a generous margin.

In the UK market spread, volatility is not confirmed to rise as expiration approaches. Using spread trading volume as a proxy for liquidity, a strategy of maximizing liquidity subject to a practical volume constraint (Strategy L) is constructed. If maximizing spread liquidity during the RAP is the investor's objective, then Strategy L is highly desirable, outperforming the even roll and stack roll strategies. In all four futures markets, by ignoring the information content in the spread volume and volatility profiles, both even and stack roll strategies produce sub - optimal roll plans.

The following practical guidelines for rolling calendar spreads are suggested by these results :

- To achieve high liquidity, confine rollover transactions to the RAP and concentrate on days with higher liquidity, which typically occur within 5 trading days before expiration.
- To maximize aggregate roll liquidity subject to transacting during more liquid RAP days, use an optimization approach similar to Strategy L.
- To minimize aggregate volatility subject to transacting during more liquid RAP days, use an optimization approach similar to Strategy V.

Limitations of the Study and Opportunities for Further Research

The findings in this study are based upon data relevant to expirations in 2016. While the stability of the relationships suggest that they are likely an expected feature of rollover markets, this surmise would benefit from further verification. Data for 2017 - present might be gathered for similar analysis. Confirmation of continuing 2016 spread behavior would support more firmly the value in applying Strategies L and V.

A second interesting extension of this study would involve testing if similar relationships involving spread volume, volatility, and liquidity can be found in other global futures markets, especially ones which are smaller or less developed. If general relationships persist broadly across global stock index futures markets, then the value to practitioners of optimizing strategies devised here will rise accordingly.

The present study is based upon analysis of day-end prices ; whereas, in practice, rolling futures positions are likely to take place intra-day. Real time testing of the optimal roll strategies would be useful to confirm the true benefits of optimized roll plans. Yet another extension of this study could be to examine related rollover strategies for index options, a more challenging topic due to the absence of a spread market. An extension to construct rolling strategies for bond futures in global futures markets might also be considered.

References

Abken, P. A. (1989). An analysis of intra-market spreads in heating oil futures. *Journal of Futures Markets, 9* (1), 77 - 86.

Adrangi, B., Chatrath, A., Song, F., & Szidarovszky, F. (2006). Petroleum spreads and the term structure of futures prices. *Applied Economics, 38* (16), 1917 - 1929.

Anson, M. (1998). Spot returns, roll yield, and diversification with commodity futures. *Journal of Alternative Investments, 1*(3), 16 - 32.

Attaqui, S., Mellios, C., & Six, P. (2011). Calendar spreads in commodity futures markets, risk premium and the convenience yield. *Bankers, Markets & Investors, 112,* 16-33.

Campbell & Company. (2014, February). *Deconstructing futures returns : The role of roll yield* (Campbell White Paper Series). Retrieved from https://www.cmegroup.com/education/files/deconstructing-futures-returns-the-role-of-roll-yield.pdf

Dunis, C. L., Laws, J., & Evans, B. (2006). Modelling and trading the gasoline crack spread: A non-linear story. *Derivatives Use, Trading & Regulation, 12* (1 - 2), 126 - 145.

Gatev, E., Goetzmann, W. N., & Rouwenhorst, K. G. (2006). Pairs trading: Performance of a relative-value arbitrage rule. *Review of Financial Studies, 19* (3), 797-827.

Girma, P. B., & Paulson, A. S. (1999). Risk arbitrage opportunities in petroleum futures spreads. *Journal of Futures Markes, 19* (8), 931 - 955.

Jones, F. J. (1981). Spreads : Tails, turtles, and all that. *Journal of Futures Markets, 1* (4), 565 - 596.

Miao, G. J. (2014). High frequency and dynamic pairs trading based on statistical arbitrage using a two-stage correlation and cointegration approach. *International Journal of Economics and Finance, 6*(3), 96 - 110.

Monroe, M. A., & Cohn, R. A. (1986). The relative efficiency of the gold and treasury bill futures markets. *Journal of Futures Markets, 6* (3), 477- 493.

Mouakhar, T., & Roberge, M. (2010). Optimal approach to futures contract roll in commodity portfolios. *Journal of Alternative Investments, 12*(3), 51 - 60.

Parkinson, M. (1980). The extreme value method for estimating of the rate of return. *Journal of Business, 53* (1), 61- 65.

Peterson, R. L. (1977). Investor preferences for futures straddles. *Journal of Financial and Quantitative Analysis, 12* (1), 105 - 120.

Rentzler, J. C. (1986). Trading treasury bond spreads against treasury bill futures - A model and empirical test of the turtle trade. *Journal of Futures Markets, 6* (1), 41- 61.

S & P Dow Jones Indices. (2016, August). *S&P GSCI® Dynamic roll methodology.* Retrieved from http://us.spindices.com/documents/methodologies/methodology-sp-gsci-dynamic-roll.pdf

Schrock, N. W. (1971). The theory of asset choice: Simultaneous holding of short and long positions in the futures market. *Journal of Political Economy, 79* (2), 270 - 293.

Simon, D. P. (1999). The soybean crush spread: Empirical evidence and trading strategies. *Journal of Futures Markets, 19* (3), 271- 289.

Slivka, R. T., & Li, X. (2010). Hedging and synthetic index funds creation in the China market. *Journal of Indexes, 13* (6), 50 - 55.

Slivka, R. T., Li, X., & Zhang, Y. (2011). Calendar spreads in China stock index futures. *Journal of Indexes, 14* (3), 42 - 48.

Slivka, R. T., Yang, J., & Wan, W. (2017). Nifty futures rollover strategies. *Indian Journal of Finance, 11*(7), 7 - 21. doi:10.17010/ijf/2017/v11i7/116563

Spurgin, R. (1999). A benchmark for commodity trading advisor performance. *Journal of Alternative Investments, 2*(1), 11-21.

Taylor, N. (2015). *Roll strategy efficiency in commodity futures markets* (Accounting and Finance Discussion Paper 15/1). Retrieved from http://www.efm.bris.ac.uk/economics/accfin_working_papers/afdp151.pdf

Wahab, M., Cohn, R., & Lashgari, M. (1994). The gold - silver spread : Integration, cointegration, predictability and ex-ante arbitrage. *Journal of Futures Markets, 14* (6), 709-756.

Yang, D., & Zhang, Q. (2000). Drift-independent volatility estimation based on high, low, open and close prices. *Journal of Business, 73*(3), 477 - 491.

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